The Future of Rough Sets

In March 2019, we asked the members of the IRSS Advisory Board to write a short contribution (two to three pages) providing some directions for future research in rough set theory. We collected 13 contributions, covering the different facets and application fields of rough set theory.
We provide here a list (in alphabetical order) of these contributions with a short summary, as a pointer to the entire collection. These contributions will also form the starting point of a round table which will be held at next International Joint Conference on Rough Sets (IJCRS19) in Debrecen, on June, 20th.

Chakraborty, M. "Rough Sets: Looking Forward"
After providing an historical perspective of RST, Mihir Chakraborty lists five points that "RST as a mathematical theory" should address: axiomatic foundation; summarization of the algebraic approaches; topology of covering rough sets; modal logic and covering rough sets; a general theorization of granules. He also points out the need for a book on rough sets for beginners.

Greco, S.; Matarazzo, B.; Slowinski ,R. "Distinguishing Vagueness from Ambiguity in Reasoning About Ordered Data through Rough Set Approximations"
The contribution of Greco, Matarazzo, and Slowinski uses the (extension of) the rough sets technology to deal with the notion of vagueness and other related concepts considered in Artificial Intelligence. The proposal uses their Dominance-based rough sets theory to distinguish between concepts such as vagueness and ambiguity. Real data often exhibits one or both of these imperfections and the authors' approach allows to introduce methodologies to current techniques of data science to improve the qualities of derived conclusions.

Janicki, R. "Directions for Future Research in Rough Sets: Optimal Approximations"
In this contribution, the problem of defining Optimal Approximations is put forward: a clear definition does not exist, but it should be based on a proper notion of similarity. Also important is the related problem of finding Optimal Approximations: it may not be feasible, hence a second-order approximation is needed.

Marek, V. "Linguistic aspects of Rough Sets and an application of Rough Sets in Analytic Processing"
The underlying idea is to bring back rough set theory to the core of computing. Some suggestions in this sense are given: to introduce Rough Sets into OLAP (OnLine Analytical Processing), which requires to provide a declarative language similar to SQL and fast data processing, for instance with a distributed handling.

Moshkov, M. "Extensions of Dynamic Programming for the Study of Decision and Inhibitory Trees and Rules"
The author describes his works in the field of decision trees and rules, envisaging that there can be connections with rough set theory which are worth to be explored in a deeper manner.

Pal, S.K "Rough Sets and Deep Learning: Some Concepts.
Rough Sets and Deep Learning are two complementary technologies that mutually reinforce each other. The author points to the potential for use both in a common setting.

Peters, G. “Some Directions for Future Research in Rough Sets”
According to the author the topics worth a deeper investigation are applications of rough sets to Dynamic Systems, Game Theory (towards a rough game theory). For the rough set community, it would also be beneficial to promote rough-set outside the usual community, creating a bridge between rough sets and other domains.

Peters, J.F. “Reflection on the future of rough sets”
Future developments in the topology of rough sets are discussed. In particular, Closure Weak topology of rough sets is proposed: advances in the topology of rough sets that takes into account the role of cell complexes in the characterization of data. This approach “will lead to a new view of indiscernibility”.

Polkowski, L. “Some remarks on the state of rough sets with a subjective view on research therein”
Some ideas for future researches and rough sets applications in new fields are given: social informatics such as surveying and making decisions out of it; to apply the generalized notion of betweenness to robotics (for navigating) and data science (for new classifiers based on partial identity).

Sakai, H “On Rough Set-based Rule Generation and Apriori-based Rule Generation: A Comparison of them and a Combination for Effective Rule Generators”
The author proposes the combination of Rule Generators: rough-set-based and Apriori-based ones to take advantage of the strengths of each one.

Skowron, A. “Interactive Granular Computing (IGrC): Rough Sets based on Judgment over Dynamic Networks of Complex Granules”
Many challenges in IGrC are related to reasoning, here called adaptive judgment, including the approximation of the complex vague concepts which could lead to extensions of rough sets to adaptive rough sets and rough sets over distributed networks of granules changing with time.

Yao, Y. “Three topics on rough set research”
Three topics for future research are proposed:

- More attention to the semantics of rough set theory;
- The interplay of rough sets and three-way decision may provide good opportunities for us to move a step further in advancing the two theories;
- It might be the time to look at possibilities of integrating three theories in which rough set theory is one of them (for instance, formal concept analysis, rough sets, and three-way decisions)

Ziarko, W. " Some Thoughts About the Future and Role of Rough Set Paradigm"
The importance of application-oriented research is stressed, which should+develop in two directions:"development of new extensions or theories [...] which are more applicable to
real-world problems” and "working on specific practical applications”.Moreover, the development of basic software for rough set on top of which build final applications is advocated.

June, 5th 2019

Davide Ciucci, IRSS President
Victor Marek, Chair of the IRSS Advisory Board

Rough Sets: Looking Forward

Mihir Kumar Chakraborty

If we consider Z. Pawlak's paper in 1982 [Rough Sets , Int. Jour. Comp. Inform. Sci. 11 (5), pp 341—356] as the origin then the theory of Rough Sets (RST) is now 37 years old. Is this age enough to attain maturity? Or what at all is meant by the word 'maturity' in the case of a subject of knowledge? There is obviously no set standard. But yet, one can say that when an area of research becomes sterile, when no interesting new results are being produced for over a reasonably long period of time the subject has grown old. Of course it has also happened in history that after quite some time the subject may resurrect. However, these are accidents, rare events.

From this angle of course, we must admit that RST is passing through its youth stage filled with enthusiasm. There are plenty of conferences around apart from the yearly event IJCRS (International Joint Conference of Rough Set). Besides, in almost all conferences on AI, Soft Computing, Data Mining, Granular Computing etc. there are invariably presented a number of papers on rough sets or hybrid of rough and fuzzy sets or something of the sort. We also have noticeable contributions on the theoretical aspects of the theory, though relatively in small quantity. So, the state of rough set research at present is not in a bad shape anyway. However, the era is still that of quantitative growth and there is nothing wrong in it; it has to be so during the starting decades of any theory. But in my view, after 37 years it is now time to look back and take a stock.

The first book on RST viz. 'Rough Sets: Theoretical Aspects of Reasoning about Data' by Pawlak published in 1991 may be considered as a prototypical embryo of the future development of the subject. It introduces the theory with an eye on possible applications, in particular on data analysis, transformation of data into knowledge, representation of and reasoning about knowledge and decision mechanism (along with algorithm). The basic ideas were so elegant and simple that immediately after the publication of the 1982 paper by Pawlak, researchers from both the fields of theory and application, enthusiastically jumped in. From the theoretical angle it was immediately realised that in the concept there was embedded the possibility of development in the directions of set theory, algebra, topology and logic. As for application, Pawlak's above mentioned book published almost ten years later after the advent of the theory showed the path [see the contents of the book as mentioned above]. This book also points at bringing the theory as close as possible to the real-life scenario. One subsection of Pawlak's book has the title "Beauty Contest", and he was not joking. The present writer remembers one of his conversations near the river Seine in Paris when Prof. Pawlak said, if one wanted to work in the field of application he/she should address a problem from real life, not an artificial problem; otherwise one may work on theory but it should be deep and nice mathematics. I consider this opinion of Pawlak as a measuring device of any hybrid mathematical theory and in particular of RST. Judging from this angle I would be happy to see that applications are addressing specific, actual and outstanding real-life problems of any field including sciences and try to solve them using RST or a hybrid of RST with other techniques. For example in the ISI (Indian Statistical Institute), Kolkata renowned researchers think that mathematical foundations of Soft Computing should be based on RST, Fuzzy Set theory, Genetic Algorithm and Neural Network. The present author, not being an expert in application, is not in a position to judge the claim but definitely appreciates the approach. But problems are to be picked up from reality.

I can, however, express more focussed view in the field of theory.

First, the foundation of RST as a mathematical theory is not yet deeply studied. It should consist at least of the following:

the philosophy, the axiomatic foundation as in axiomatic set theory, the logic/logics on which RST is based or that emerge from the theory and category theoretic studies.

Second, undoubtedly there has been extensive studies on the algebraic aspect. Most probably this is one single aspect in which quite significant research has been carried out. The time perhaps has arrived to summarise. I have in mind the way similar to that classical set theory is related with Boolean algebra or Intuitionistic Set theory with Heyting Algebra, Fuzzy set theory with MV algebra etc.

Third, although the topological aspect of RST has been quite seriously investigated by some important researchers, it still deserves deeper investigations particularly with respect to covering based rough sets.

Fourth, a related issue is the connection with modal logics. It is known that modal logic systems admit topological (neighbourhood) semantics. On the other hand, coverings are neighbourhood systems or generate some. The trio: covering rough sets. Neighbourhood systems and modal logics from an interesting triangle with mutual interrelation which should constitute a fascinating area of research not only a theoretical one but bearing great potentiality of applications. Granules are not generally disjoint , so equivalence classes are not usually available in real life.

Fifth, a general theorization of granules that consist of indistinguishable non-identicals need to be developed. Such a theory would have the potentiality to address a long list of issues both conceptual as well as practical.

There are surely other directions too in my view and of others among which only some have been mentioned above.

I would like to end this short article with expressing the pressing need to have a few text books. As a parallel, we may consider fuzzy set theory (FST) founded by L.A.Zadeh through his paper 'Fuzzy Sets' in 1965. So it has lived 54 years, of course a longer life than RST and it is well known that there are many overlaps between FST and RST. But while FST has achieved a greater popularity, RST is still lagging behind. One of the reasons is undoubtedly that in FST there are several very good text books while there is in fact none on RST in the international language. [The Chinese of course have published some for the readers of that language.] By such a book I do not mean compilation of articles by different authors, There are quite a number of such volumes edited by very important and efficient persons. Rather I mean books that may be recommended for reading as reference, that will introduce the basics of the theory , show its significant applications in various fields and indicate the frontier areas of research . I have faced problem while offering courses on RST in many places. Below is mentioned some books of this category other than the book by Pawlak, but one can see that these do not intend to introduce the subject to a new learner [except perhaps the book by Polkowski which does not deal with the areas of application], though they might be good for researchers. The rough set community should take up this point very seriously. I think there would be no problem with publishing.

Few Books on Rough Set Theory:

Lech Polkowski, Rough Sets : Mathematical Foundations, Springer , 2002

Piero Pagliani and Mihir Chakraborty, A Geometry of Approximation : Rough Set Theory: Logic, Algebra and Topology of Conceptual Patterns, Springer, 2008

Stepiane P. Demeri, Ewa Orlowska, Incomplete Information : Structure, Inference, Complexity,

Springer, 2010

Distinguishing Vagueness from Ambiguity in Reasoning About Ordered Data through Rough Set Approximations

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We present our vision of a future perspective of rough set theory and its applications to reasoning with imperfect knowledge. We claim that in this process one cannot ignore some important features of data related to their ordinal character and to the distinction between vagueness and ambiguity.

Keywords: Imperfect knowledge, Vagueness, Ambiguity, Rough sets, Ordinal data, Dominance-based Rough Set Approach, Pawlak operator

Ordinal properties of data are among basic features to be taken into consideration when analysing information to induce relevant knowledge. Indeed, the basic idea of relationship between attributes is generally expressed through the concept of correlation for which the increase of a value in one dimension is linked with the increase (or the decrease, in case of negative correlation) in another dimension. Therefore, the same basic concept of correlation requires that the data are expressed on an ordered scale, because otherwise, the underlying notion of increase or decrease would be meaningless. Consequently, any methodology of data analysis remains inadequate if it is not able to handle ordinal data. On the basis of this observation, and paying special attention to data expressing preferences, consideration of ordinal properties of the data has been integrated in rough set theory[7,8] through the Dominance-based Rough Set Approach[3] (DRSA). DRSA permitted to introduce concepts, approaches and methodologies from data mining, knowledge discovery and artificial intelligence to the domain of decision analysis and preference handling. The benefits of this operation can be summarized in the two following points:

1) from the point of view of information to be processed, the focus is moved from direct elicitation of some abstract parameters of a preference model by the decision maker, such as tradeoffs between attributes, to processing of some holistic decision examples provided by the decision maker, such as assignments of some prototypical objects to preference ordered classes or pairwise preference comparison of some objects on which the decision maker has matured a conviction;
2) from the point of view of knowledge stemming from the data analysis, its representation has passed from a preference model expressed through analytically formulated utility functions or outranking relations to logical statements in terms of "*if ..., then ...*" decision rules relating in an intelligible way evaluations on selected attributes with overall classification or preferences.

Comparing to usual decision support methodologies, these benefits ensure an easier communication between the decision maker providing the data and the data processing method. They reduce the cognitive burden of the decision maker that finds in DRSA and its decision rules a user-friendly tool for reasoning about her/his preferences and their impact on decision making. In this perspective it is also worth stressing that the natural language in which the decision rules are expressed implies that using DRSA the DM is not required to have a background in decision theory in order to interpret the result of the decision support methodology.

More in general, observe that the possibility of considering ordinal properties of data by DRSA, permits to apply rough set theory in any context where the ordinal character of data is relevant. According to the initial reflection on the necessity of considering ordinal data when the concept of correlation is involved, it follows that DRSA is applicable for any kind of data to be analysed.[1]

On this ground, consideration of different kinds of data and knowledge imperfection becomes fundamental for future development of theory and practice of rough sets. More precisely, following,[4–6] we can distinguish between the following kinds of data imperfection:

- vagueness, due to imprecise and uncertain knowledge, related to 'a priori' epistemic evaluation of the credibility of one concept, and
- ambiguity, due to granularity of knowledge and coarseness of information, related to 'a posteriori' approximation of the same concept by means of granules of knowledge.

The basic idea of the recently introduced methodology permitting to distinguish vagueness from ambiguity in the rough set theory[6] is the following. Each concept in the initial information is characterized by a vagueness represented by an orthopair[2] being a pair of disjoint sets in the universe of knowledge, such that the first set contains all the objects that 'a priori' are considered as surely belonging to the concept, while the second set contains all the objects that 'a priori' surely do not belong to the concept. The knowledge obtained from the initial information is characterized by an ambiguity represented by means of the Pawlak operator, being a rough approximation of the orthopair representing the considered concepts in terms of another orthopair composed of the lower approximations of the two sets in the original orthopair. This methodology can be applied in the original rough set context where there is no consideration of ordinal properties, but it is clear that it can give the most promising results when applied to ordinal data through an extension of DRSA that incorporates the Pawlak operator.

In conclusion, we believe that the most prospective development directions of rough set theory and application rely on the conjoint consideration of ordinal properties of data (in particular, when data concern preferences, but not only), and the distinction of vagueness and ambiguity.

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Directions for Future Research in Rough Sets: Optimal Approximations

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Rough sets provide excellent mathematical background for dealing with *approximations*, especially when objects of consideration are complex mathematical structures such as sets, multisets, relations, etc. Human beings are actually able to use approximative data and values in daily lives and various stages of decision making processes. However for AI and machine learning applications, a story is very different. Dealing with exact or all data and values is often not feasible, but even *defining* what is an acceptable (or optimal) approximation, not to mention finding it, is far from obvious in much of the cases.

The concept of approximation has two different intuitions in mathematics and science. The first one stems from the fact that all empirical numerical data involve errors, so in reality we do not have an exact value x, but always an interval $(x-\varepsilon, x+\varepsilon)$, i.e. the upper approximation and the lower approximation of data. Plain rough sets exploit and generalize this idea for more general data structures.

The second intuition can be illustrated by the *least square approximations* of points in two dimensional plane.

Here we know or assume that the points should be on a straight line and we are trying to find the line that fits the data best. In this case the data have a clear structure (points in two dimensional plane, i.e. a binary relation) and should satisfied a desired property (be linear). Clearly this is not the case of an upper, or lower approximation in any sense. However this approach assumes that there is a well defined concept of a *metric* which allows us to minimize the distance, and this concept is not obvious, and often not even possible for non-numerical objects.

It appears that for many applications, the rough sets need yet another, *third* approximation, that could be interpreted as an *optimal approximation* or *property-driven approximation*. In general, we can only require that an optimal approximation is a subset of upper approximation and a superset of lower approximation, but even nonempty intersection with a set it approximates may not hold in some cases.

Some initial models of such "optimal" approximations have already been proposed, cf. [1, 2, 3, 4, 5, 7], but in general the problem can be regarded as almost open. While the definitions of both lower and upper approximations are intuitively solid and rather indisputable, it is not clear how "optimal" approximation should be defined. Most likely several different reasonable approaches are possible. To define any concept of "optimal" approximation, we need a proper definition of *simultaneity*, and there are many possible choices [6]. It is also not clear what kind of simultaneities fit best rough sets approach [2]. Moreover, simultaneity may be defined for the elements - as in [5], or sets - as in [2, 3], or both - however there are apparently no known results for this case. It also turns out that even for very simple similarity measures and rather simple desired properties the problems often may become NP-complete, so some second level of approximation might be needed [2].

Often finding "optimal" approximation may not be feasible. Consider the following problem: we have a set of data that have been obtained in an empirical manner. From the nature of the problem we know that the set should be partially ordered, but because the data are empirical it is not. In a general case, this relation may be arbitrary. What is the 'best' partially ordered approximation of an arbitrary relation and how this approximation can be computed? An answer to that question may not be feasible, as the problem is NP-hard even for very simple specially designed similarity measure [2]. On the other hand, if we skip 'best', and be satisfied with just 'partially ordered approximation', relatively efficient solutions can be found [1]. We can first compute *acyclic kernel*, which is rough sets type property-driven lower approximation, and then *transitive closure* of the result of first step, which is rough set type property-driven upper approximation. Or, we can do the same steps but in the opposite order. The outcome is a partial order that, in both cases, can be seen as a reasonable partial order approximation of the initial data. Of course, the problem is more difficult for more complex properties and data structures. These kind of "property driven" rough sets type approximations was first proposed in [7].

I believe adding the third, "optimal" or "property-driven", approximation would substantially increase potential application of rough sets approach.

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Linguistic aspects of Rough Sets and an application of Rough Sets in Analytic Processing

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In this document we outline some tasks that, in our opinion need to facilitate the use of Rough Sets in a variety of tasks, in particular used in Machine Learning, and more generally, Artificial Intelligence.

We do *not* introduce the basic concepts on which rough set area is based, as we expect the reader to be familiar with them. A significant number of papers, starting with the classical paper of Zdzislaw Pawlak, contain such information.

Here we focus on tasks that need to be done to introduce Rough Sets into the *mainstream* of applications of Computer Science (specifically: online analytic processing usually known by its acronym, *OLAP*, one of mechanisms for handling some aspects of "big data"). We discuss these tasks and the use of rough sets in them in a number of short sections that describe the associated issues.

1 Standard Description Language, handling of statistical functions, and sublattices.

While complex object-based languages may be conceived for the task at hand, the simplest language for description of the objects in rough sets could be the language consisting of grounded atomic predicates. To give a trivial example, the object "data_marek" could be

$$< lname = marek, fname = victor, city = lexington >$$

If this resembles the reader the so-called object-description language, it is not an accident, as that formalism is closely related both to logic and databases, that provide intuitions related to rough sets.

We observe that it would be also natural to have a distinguished second-order predicate "similar" (denoted $\simeq$) to denote the equivalence relation of the described rough set, for instance

$$< lname = marek, fname = victor, city = lexington > \simeq$$
$$< lname = marek, fname = victor, city = danville >$$

With this formalism one could build-in the standard axioms of equivalence relation (reflexivity, symmetry, transitivity) of $\simeq$ without introducing them explicitly.

It should be clear (at least coming from this writer) that we envision a *declarative* language that could, like SQL, handle all three main tasks of data processing: data definition, data manipulation, and data querying. Hints from the syntax of declarative languages such as SQL may be useful in the design of such language. The differences (related mostly to the role of the "equivalence predicate", $\simeq$, and associated statistical functions, for instance the computation of "interior" and "closure" operations and the use of size functions (for instance: the "size of interior of the class of object o") will facilitate applications such as one described in Section 3.

Like SQL, the putative declarative language needs to provide extensive statistical functions and set functions (including size computation and size comparison constructs). A rich constraint fragment (extending the constraints available in SQL) needs to be implemented, as well. Search via regular expressions must be available. Standardization of the language for Rough Sets handling will eventually allow for building libraries for interaction with imperative languages (JAVA, Python, and others).

2 Distributed processing of Rough Sets

The applications we have in mind (see the next section) relate to large databases (the intuition, as the reader certainly realizes, comes from processing in real time of data associated with purchases of goods). Such systems are characterized by large volumes, arriving in real time, and the need for fast reaction to the data. From the perspective of the defense/expansion duality discussed in the next section, the processing can not be limited to local systems, but rather to the networked large repositories of data. Given that the data arrives in real time and decisions/processing must also happen in real time, there is need to return the output as the customer is still available for interaction. For that reason fast algorithms processing changing data are needed. A natural solution is, of course distributed handling of the data in a Hadoop2 or SPARK mode.

3 One industrial application

Commercial applications of Rough Sets, for instance in the industries such as grocery vending, and other mass-market businesses can not use Rough Sets technology without libraries of functions for a variety of statistics. To give an example of the usefulness of such data, let us look at grocers. They need to both *defend* their share of the market and attempt to *expand* their share of the market. In the first of these tasks, incentives must be provided to customers to *continue* to purchase goods. This is done in North America by means of coupons sent to customers to bind them to the vendor. Such coupons are provided at check-out time, or sent to the customers via mail, e-mail, or other channels. Data on 'who buys what' are needed for such task, and suitably chosen queries of the type discussed above can be used for that purpose.

The queries used for expansion again use statistics, by allowing to compute the items that *normally* accompany purchases of some goods. Again, coupons are sent to (or printed at checkout for) customers that need to be enticed to make purchases (the reasoning is that they buy the items elsewhere). Again, data (but this time on 'who would likely be interested in what') can be computed using statistical functions.

It is easy to see that the queries discussed in previous two paragraphs are related to sublattices of the partitions generated by the rough set consisting of data collected by vendors.

In database perspective, we see that we are using rough sets to solve an OnLine Analytical Processing problem. Of course, additional parameters (such as a threshold of similarity), and possibly other designer-supplied quantities are required, but an interesting and societally important problem is solved.

We observe that related issues were discussed in the contribution of Slezak et.al, [1].

Reference:

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Extensions of Dynamic Programming for the Study of Decision and Inhibitory Trees and Rules

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The conventional dynamic programming algorithms for optimization problems include a structure of sub-problems of the initial problem, a way to construct a solution for a sub-problem from solutions of smaller sub-problems, and solutions for the smallest sub-problems. Such algorithms return only one solution.

In books [1, 2], we consider extensions of the dynamic programming approach that allow us (i) to make multi-stage optimization relative to a sequence of criteria, (ii) to describe the whole set of solutions or its essential part, (iii) to count the described solutions, and (iv) to construct the set of Pareto optimal points for a bi-criteria optimization problem.

In [1], we apply this approach to the study of decision trees and rules for conventional decision tables with single-valued decisions. In [2], we generalize the considered approach to the case of decision tables with many-valued decisions. We also study inhibitory trees and rules which instead of expressions “decision = value” use expressions “decision ≠ value”. Inhibitory trees and rules can, sometimes, describe more information derivable from a decision table than decision trees and rules.

For decision and inhibitory trees and rules, we designed algorithms (i) for multi-stage optimization relative to a number of criteria (for example, depth, average depth, and number of nodes for decision trees), (ii) for counting the number of optimal trees and rules, and (iii) for bi-criteria optimization (such algorithms construct the set of Pareto optimal points for two criteria).

The majority of problems related to the optimization of decision and inhibitory trees and rules are NP-hard. The considered algorithms have exponential time complexity in general case. However, we described classes of decision tables for which these algorithms have polynomial time complexity.

In book [1], we consider the following applications of the created tools.

For the decision trees, the applications of multi-stage optimization approach include the study of totally optimal (simultaneously optimal relative to a number of cost functions) decision trees for Boolean functions, improvements on the bounds on the depth of decision trees for diagnosis of constant faults in iteration-free circuits over monotone basis, computation of minimum average depth for a decision tree for sorting eight elements (a problem that was open since 1968), study of optimal reducts and decision trees for modified majority problem, and designing an algorithm for the problem of reduct optimization. The applications of bi-criteria optimization approach include the comparison of different greedy algorithms for construction of decision trees, analysis of trade-offs for decision trees for corner point detection (used in computer vision), study of derivation of decision rules from decision trees, and a new technique called *multi-pruning of decision trees* which is used for data mining, knowledge representation and classification. The classifiers constructed by multi-pruning process often have better accuracy than the classifiers constructed by CART [3].

For the decision rules and systems of rules, the applications of multi-stage optimization approach include the study of decision rules that are totally optimal relative to the length and coverage (have minimum length and maximum coverage simultaneously), investigation of a simulation of a greedy algorithm for the construction of relatively small sets of decision rules that cover almost all objects (rows), and comparison minimum depth of deterministic and nondeterministic decision trees for total Boolean functions. The set of Pareto optimal points is used to measure the quality of multiple heuristic methods for the construction of systems of decision rules.

In book [2], we also consider a number of applications.

For the decision and inhibitory trees, the applications of the multi-stage optimization approach include the study of the minimum depth, minimum average depth, and minimum number of nodes in decision trees for sorting n = 2, …, 7 elements among which there are, possibly, equal elements, and the study of totally optimal (simultaneously optimal relative to a number of cost functions) decision and inhibitory trees for decision tables with many-valued decisions obtained from decision tables from the UCI ML Repository [4] by removal of some conditional attributes. The applications of the bi-criteria optimization approach include the comparison of 12 greedy heuristics for the construction of decision and inhibitory trees for decision tables with many-valued decisions as algorithms for single-criterion and bi-criteria optimization, the study of two relationships for decision trees related to the knowledge representation – number of nodes versus depth and number of nodes versus average depth, and the study of a new technique called *restricted multi-pruning of decision trees* which is used for data mining, knowledge representation, and classification.

For the decision and inhibitory rules and systems of rules, the applications of the multi-stage optimization approach include the study of decision and inhibitory rules that are totally optimal relative to the length and coverage, and the investigation of a simulation of a greedy algorithm for the construction of relatively small sets of decision rules that cover almost all rows in decision tables with many-valued decisions. The applications of the bi-criteria optimization approach include the comparison of 13 greedy heuristics for the construction of decision rules from the point of view of single-criterion optimization (relative to the length or coverage) and bi-criteria optimization (relative to the length and coverage).

The considered algorithms are not for industrial applications. They can work only with at most medium-sized decision tables. However, they can give us essentially more information about such decision tables than greedy algorithms: we can construct optimal decision and inhibitory trees and rules relative to different criteria, study relationships between two criteria, etc. The considered approach can be useful for investigations in different areas that study decision rules and trees, in particular, in rough set theory.

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Some Directions for Future Research in Rough Sets

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Applications of Rough Sets to Dynamic Systems

In rough sets boundary contain the objects that cannot be assigned to single set. In general, this is independent of the reasons why the assignment to one and only one set fails and could include missing features or contradicting data. Missing or contradicting data could be cause be, e.g., faulty sensors recording the data.

Dynamic systems, i.e., systems the changing parameters, moving objects etc. over time, can be observed virtually everywhere. Let us simply change our first sentence a little bit to: In rough sets boundary contain the objects that cannot be assigned to single set yet. This might indicate the potential of rough sets for dynamic systems. At the time of analysis, objects in boundaries lack of information that may become available at a later time.

There are already some papers in this field but given omnipresence and importance of dynamic systems, there still seem to still great potential and need for further progress in this field.

Applications of Rough Sets to Game Theory

Presently, probably most of the papers on rough sets and game theory are in the field of game theoretic rough sets. In this field game theory is used to determine the optimal assignment of the objects to the upper and lower approximations of the sets. So, game theory is applied as optimization method like in many other fields.

However, given the fundamental idea of rough set, separating the objects with clear memberships (lower approximation of a set) from objects with unclear memberships (boundary of a set) may be very helpful for games also. For example, in a game with two players, one player may know a subset of the strategies/pay-offs of the second player while it has no information on another subset of the strategies/pay-offs of its opponent.

There already some papers applying rough sets to game theory. However, considering the long and rich history of game theory, it might be fruitful to further investigate the potential of rough sets to game theory, i.e., integrating rough concepts into game theory, towards a rough game theory.

Applications of Rough Sets to Real Life Problems

Over the past decades, since its introduction, rough sets have made an impressive journey. Although mature levels have been obtained in many parts of rough set theory it is still young field with much potential in particular outside its core community.

The rough set community is constantly growing however still small. This is great since many researchers know each other personally. However, given the potential of rough sets, applications to other scientific fields (see above to, e.g., game theory) and applications to real life problems (see above to, e.g., dynamic systems) rough sets is still selling itself at less than fair value.

Therefore, it might be worth to think about strategies to more actively promote rough sets beyond the borders of its community. In particular, even more domain experts other than in rough sets, e.g., experts in social and natural sciences or in engineering etc., should be introduced to rough sets. Such an extended bridging between rough sets and other domains has been mutually be very beneficial for the rough sets community as well for the non-rough sets domain experts already. Therefore, intensifying this exchange would continue to the success of rough sets and will lead to a win-win situation for rough sets and the non-rough sets domains.

REFLECTIONS ON THE FUTURE OF ROUGH SETS

JAMES F. PETERS

The future of rough sets will include advances in the topology of rough sets that takes into account the role of cell complexes in the characterization of data that provide a basis for information tables. A cellular view of data takes into account the work P.S. Alexandrov (1935) and J.H.C. Whitehead (1939) on closure finite weak (CW) topology. For simplicity, I consider only a planar view of data. There are three types of cells to consider, namely, 0-cells, 1-cells and 2-cells.

In a planar view of data, a datum is a zero-cell (called a vertex CW topology). A pair of data that are related are one-cells (0-cells with an edge attached between them). And a trio of related data are 2-cells. A rough 2-cell has a triangular shape whose interior is filled with data that are in the interior of the 2-cell and which are ″touched″′ (correlated) with the vertexes of the 2-cell.

In a **CW topology of rough sets**, a lower approximation is a 2-cell with a nonempty boundary, which is a collection of cell complexes with nonempty intersection, *i.e.*, a collection of cells that can be classified with certainty. This view of lower approximation carries forward the original view of the lower approximation of a nonempty set introduced by Z. Pawlak in his introduction to rough sets (TRS I, 2004, 1-58). In keeping the CW topology paradigm, the traditional closed (open) set view of collections of data is replaced by the closure view of collections of data. In that case, a lower approximation of a set is the closure of that set that includes both the boundary as well as the interior of the set. And the set itself is a cell complex.

The advent of a CW topology of rough sets will lead to a new view of indiscernibility. Traditionally, a pair of data belong to an indiscernibility relation, provided the data have matching descriptions (J.W. Gryzmala-Busse, TRS I, p. 81). In a CW topology of rough sets, a pair of data are indiscernible, provided the data belong the interior of the same 2-cell, independent of their descriptions. In other words, the geometry of this form of rough sets overrides the matching descriptions requirement of traditional rough sets.

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Rough Set and Deep Learning: Some Concepts

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Abstract:

Features of granular computing and relevance of rough sets is stated. This is followed by the characteristics of deep learning and deep architecture, and significance of deep convolutional neural networks. Finally, some sketches on granulated deep learning, and rough deep framework through evolution and learning of granules are provided.

Granular Computing: Features and Rough Sets

Granulation is a basic step of human cognition system. It is a process like self-organization, self-production, morphogenesis, Darwinian evolution that are extracted from natural phenomena. It may be viewed as a process of natural clustering, i.e., replacing a fine-grained universe by a coarse-grained one, more in line with human perception. Clusters or segments so formed by granulation (natural clustering) are called granules. In other words, granules evolve through information abstraction and derivation of knowledge from data in the process of granulation.

Since granulation leads to information compression, processing based on the compressed information, rather than the individual data points, may lead to gain in computation time. This makes Granular computing (GrC) a good candidate for data mining and knowledge discovery.

Rough set theory that deals with the concept of a set defined over a granulated domain has proven to be effective in GrC research. Here the set is approximated in term of granules from inside and outside (lower and upper approximations). This inexact definition of set signifies the incompleteness in knowledge about the universe, thereby resulting in uncertainty in the system. Minimization of the uncertainty (incompleteness in knowledge) played a pivoted role in image/ video processing [1], pattern recognition [2], and data mining, among others. Concept of lower/upper approximation has also been used as information granules in designing various artificial neural network (ANN) models [3] and unsupervised object tracking [4].

Deep Learning and Architecture: Concepts and Issues

Machine learning (ML), a branch of artificial intelligence (AI), basically means learning patterns from examples or sample data. Here the machine is given access to the data and is asked to learn from it. The data (or examples) could be labeled, unlabeled, or their combination. Accordingly, the learning could be supervised, unsupervised or semi-supervised. Artificial neural networks (ANNs) that have the ability to learn the relation between input and output from examples are good candidates for ML. ANNs enjoy the characteristics like adaptivity, speed, robustness/ ruggedness, and optimality. In the early 2000s, certain breakthroughs in multi-layered neural networks (MLP) facilitated the advent of deep learning. Deep learning (DL) means learning in depth in different stages [5]. DL is thus a specialized form of ML which takes the latter to the next level in an advanced form. This is characterized by learning the data representations, in contrary to task-specific algorithms.

Deep Learning algorithms/ networks are inspired by the structure and function of the human nervous system, where a complex network of interconnected computation units (nodes) works in a coordinated fashion to process complex information. In order to extract the complex representation from rich sensory inputs, human information processing mechanisms suggest the need of deep (learning) architectures [6]. Convolutional neural network (CNN, or ConvNet) [7] represents one such deep architecture which is most popular for learning with images and video.

Deep learning (DL) has dramatically improved the state of the art in object recognition [6], among other applications. However, since DL relies on sample data (or previous experience), the learning performance depends on the number of such samples. Larger the number is, more accuracy is the performance. Today, we have abundant data; so DL has become a meaningful choice. DL often requires hundreds or thousands of images for the best results unlike the conventional (Shallow) learning. Therefore, DL is computationally intensive and difficult to engineer. It requires a high-performance GPU (Graphical Processing Unit).

Granulated Deep Learning and Rough Deep Framework: Some Sketches

While deep learning is a computationally intensive process and the aforesaid granular computing paradigm, on the other hand, leads to gain in computation time, **it may be appropriate and logical to consider their integration judiciously so as to make the deep learning framework efficient in terms of computation time requiring only CPU**.

Recently, an attempt has been made in this line where rough set theoretic spatio-colour granulation in convolution layer enables CNN based deep learning framework speedy motion detection and moving object recognition [8]. Here instead of scanning the entire image pixel by pixel in the convolution layer of DL, one jumps over the granules only. For a 32x32 image with N granules, sliding the filter is done only N times instead of over 32x32 pixels, where N<< 32x32. Hence a *significant speed up* is observed, compromising some accuracy. The concept needs further investigation.

One may further note that, DL is basically an abstract concept. Although significant research is going on for formulation of DL algorithms in neural network paradigm, one may consider the design of DL architecture in rough set theoretic framework. For example, the learning mechanism in Rough set theoretic DL framework may have several steps or layers to learn granules (that are evolved through information abstraction and derivation of knowledge from data), and their representations in terms of rough lower and upper approximations. Learning the size and shape of lower/upper regions and/or information granules, thus evolved in different layers, would enable better structural representation of the data, the patterns therein, and hence the derivation of knowledge. In this way, uncertainty arising from granularity in data, as well as the computation time in decision-making would also get reduced.

The use of granular flow graph [4] for knowledge representation and updating, and rough filter [9] in successive layers may be considered to enrich the said framework. Granular flow graph maps the decision-making paths in terms of granular information. Its updating may result in in-depth learning of the input patterns.

Conclusions

Rough set and Granular computing (GrC) are proven technologies for knowledge mining and discovery in large data sets. They have characteristics like dimensionality reduction, uncertainty analysis, and gain in computation time. Deep learning (DL) and Big data analytics (BDA) has recently drawn the attention of researchers and practitioners because of its promising role in several fields, including commerce and business, biology, medicine, public administration, manufacturing, banking, and education. DL has dramatically improved the state of the art in object recognition, among many other applications. However, DL requires hundreds or thousands of images (samples) for the best results unlike the conventional (Shallow) learning, so it is computationally intensive and sometimes difficult to engineer. Some thought to overcome these are outlined here. These include the concepts of:

- Granulated deep leaning by incorporating granulation in convolution layer of DL network

- Rough DL framework consisting different layers, instead of using neural net paradigm, where granules of various sizes and shapes evolve in different stages and are learnt; thereby providing a better structural representation of the data. Use of rough filter and granular flow graph may be explored to enrich the knowledge extraction and learning the data representation in terms of lower/upper approximations.

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A subjective view on rough sets: history, present day, perspectives

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1. Asked by the Acting Chairman of the Advisory Board of IRSS, Professor Victor Marek, to give my opinion on the issues adressed in the title of this note, I am venturing my subjective view on rough sets. As one witnessing a large span of rough set history beginning from 1992, I have seen some successes of this theory. I wish to dedicate this short note to the memories of Professors Helena Rasiowa and Zdzisław Pawlak. Professor Helena Rasiowa, as it were, '*took me in the palm of Her hand*', which resulted in my occupation with rough sets and Professor Zdzisław Pawlak accepted my decision and included me into his group whose members worked on rough sets and similar topics like information systems already a couple of years by then.

2. My experience in mathematics determined my early look on rough sets. I valued the logical structure of this theory allowing for formal reasoning. At the instigation of Professor Pawlak, I looked at the topological aspects of rough sets. It was obvious that approximation operators defined within rough set theory are the interior and closure operators in the partition topology induced by indiscernibility classes (still there appear authors pursuing that topic) and in order to obtain new results one had to follow the advice by Stan Ulam: '*go to infinity first*'. Following this advice, I defined topological spaces of rough and almost rough sets which were proved to be completely metrizable and in the finite case even compact (cf., Bull. Acad. Polon. Sci. Math. resp. Sci. Tech. 1993-94 as well as my monograph 'Rough Sets. Mathematical Foundations'). It may sound anecdotal, but Professor Pawlak wanted to make sure that those results are true, so they sent the draft to someplace somewhere to somebody for a review. That somebody had only one negative remark: that in that draft there were no quotations of other papers on that topic and the only quoted author was myself. This testifies about the novelty of the rough set approach. As an application, I proposed the Approximate Collage Theorem (cf., Pal, Skowron (ed.): 'Rough - fuzzy Hybridization'). The book edited by Pal and Skowron is an example of attempts at merging rough sets with other paradigms. Other example is the monograph 'Rough-Neural Computing' edited by Pal, Skowron

and myself. Many works on those forms of hybridization came from the Kolkata ISI. The idea of exploring rough set ideas in hybridization with other theories or paradigms was realized with Professor Andrzej Skowron in about 25 published research works. Those works resulted inter alia in rough mereology (IJAR, 1996-7), an extension of mereology to the notion of a part to a degree, rough set ideas in mathematical morphology, in many agent systems, control theory, and in formal grammars (joint works with Gheorghe Paun). That path of research brought some criticism from some by then influential rough set society figures but I am still convinced that it was and is a sound way of researching rough sets. This is borne out by many research works on algebraic structures induced by rough sets and on logical structures inspired by rough sets ideology. There are now many works on 'intertopics' like reduction methods, 'three way decisions' (by the way, rough sets from the beginning were 'three way decisive'). My work has been concerned with rough mereology (called also by Varzi in Stanford Enc. Philosophy: the 'fuzzified mereology') (cf. my monograph 'Approximate Reasoning by Parts') with applications to Granular Computing (cf., the monograph 'Granular Computing in Decision Approximation' (with P. Artiemjew)) in which the formal definition of a granule, based on rough mereological partial containment, given by myself (cf. GrC 2005, at Tsinghua U. in Beijing) has been tested in classification problems. Rough mereology was applied in Behavioral Robotics toward strategies for navigation by teams of robots (see the last issue of TRS for my chapter on Spatial Reasoning). I mention this not to invite anyone to work on those topics but to convince ourselves that there are probably many works on rough sets, besides already done application oriented works, which may offer venues for applications in domains nowadays of interest.

3. From 1992, rough set researchers started to organize workshops and conferences on their own. I remember the workshop in Kiekrz near Poznań organized by Professor Roman Słowiński and his Poznań colleagues in September 1992. I met there for instance Professors Inka Rauszer and TY Lin in addition to many Polish researchers, many of which left early the rough set research community. In 1998 in Warsaw I organized as the acting and coordinating organizer the first large conference for which I proposed the acronym RSCTC (Rough Sets and Current Trends in Computing). It was the realization of my deep belief that rough sets have future in cooperation with other paradigms and that solely rough sets dedicated meetings would not have any great future. RSCTC'98 attracted many leading researchers among them Professors Lotfi Asker Zadeh, Solomon Marcus, Rakesh Agrawal and of course all active at that time Polish researchers. RSCTC was live for a few years and then it was split into a number of conferences like RSKT, etc. Finally a few years ago, those four conferences were reunited into IJCRS. The 2016 conference in Santiago de Chile boasted of 109 submissions of which about one half were included into Proceedings. In 2017, in the year of 35th anniversary of the 1982 announcement by Zdzisław Pawlak of the idea of a rough set, I organized with help of my colleagues from University in Olsztyn, Poland, the IJCRS 2017 which had 130 submissions of which majority was accepted making two volumes of proceedings to the bulk of about 1500 pages. We introduced a few novelties,

e.g., the allowance of lengthy papers up to 20 pages, we gave up on the principle that quality means the rejection of about one half of the submissions, etc. But 2018 and 2019 conferences have had about 60+ submissions. One reason at least for Poland can be the fact that the new law regulating the scientific life greatly reduces the role of conferences; I do not know if it may be the case for some other countries.

4. What is lurking for rough sets in future? It is of course difficult to foresee. In my opinion, the main problem is that rough set conferences are converging to the area of relatively small workshops. Nowadays, rational and formal models are dominated by heuristics, the good example is deep learning and it seems that rough sets will become either a theoretical small area like many other topics or they find some niches with an application potential which is already demonstrated by works on medical imagery and pattern recognition emerging from Kolkata, Chengdu, Chongqing and other centers of rough set research. The important factor is biology: the generation which exported rough set theory from local Polish ground to the world and for many years provided a flux of research results in journals and a plethora of conferences is slowly finishing their work and careers and I see that for instance in Debrecen's IJCRS 2019 where there are almost no Polish participants (maybe one of reasons was explained earlier in this text). I would like to advise in order to strengthen the impact of conferences, to extend the name IJCRS in order to include other concept approximation paradigms in order to broaden the scope and possibly attract more participants. Other problem I would like to point to is reviewing. Of course, reviewing is a weak point today in many areas. In older days, to evaluate a paper in the negative, the reviewer was obliged to point to errors and prove that they were errors; today we meet with reviews in which the reviewer simply states 'I don't understand that' and this is the cause for rejection; we could point to one sentence reviews. Rough set conferences should eliminate such behaviors. In our IJCRS 2017 conference, we clearly adopted the principle that ambiguous evaluations would be re-evaluated by members of PC. It may be helpful to introduce the rule that final evaluations of submissions are subject to confirmation by a committee(s) composed of members of AB or SC. This may increase the trust in integrity of reviews. The role of conferences may be diminishing and the greater may be the role of journals. Here, rough sets are at loss as witnessed by TRS whose issues appear less and less often. In those years of my participation in rough set research, I never got from anyone any preprint of results, it seems that the society is highly polarized and atomized on the verge of rivalry rather than cooperation, maybe IRSS could open a forum where people could send their preprints to acquaint themselves with preliminary results of research by others. This of course will require a high level of integrity and of mutual confidence. But if it turns out to be a moral victory then the interesting results would come.

Finally, looking back, I would like to say that I feel happy I could work with exceptional people on this new by then idea of a rough set and help to make it a recognized paradigm.

On Rough Set-based Rule Generation and Apriori-based Rule Generation: A Comparison of them and a Combination for Effective Rule Generators

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We have been coping with rule generation from table data sets, and studied two important frameworks. The first is rough set-based rule generation [4], and the second is Apriori-based rule generation [1]. We are also interested in information incompleteness in table data sets [2, 5, 3], and proposed Apriori-based rule generation for Non-deterministic Information Systems (NIS-Apriori) [7]. The NIS-Apriori algorithm is implemented in SQL, and we term it 'NIS-Apriori in SQL' [6].

This note reconsiders the role of rough set-based rule generation and Apriori-based rule generation for realizing more effective rule generator. Each of them can generate rules from tables like the following ψ (Deterministic Information Systems), however each rule generation technique has its original characteristics.

OB	*doors*	*persons*	*maintenance*	*acceptability*
x_1	2	2	*poor*	*poor*
x_2	2	4	*poor*	*normal*
x_3	2	4	*good*	*good*
x_4	3	4	*good*	*normal*
x_5	4	4	*normal*	*good*
x_6	4	7	*good*	*normal*

In rough sets, Proposition 1 connects the concept of consistency with the inclusion relation of equivalence classes. The problem to obtain consistent implications (at the beginning of rough set research, a rule seemed to be a consistent implication) is translated to the detection of inclusion relations. We consider this proposition will be the origin of several rough set-based frameworks.

Proposition 1. *[4] In DISs, the following two conditions are equivalent:*
(1) An object $x \in OB$ is consistent *for CON and Dec.*
(2) $[x]_{CON} \subseteq [x]_{Dec}$.

In Apriori-based rule generation, the purpose is to detect each implication τ satisfying $support(\tau) \geq \alpha$ and $accuracy(\tau) \geq \beta$ for the specified $\alpha,\ \beta\ \in [0, 1]$ [1]. The Apriori algorithm is well-studied for handling transaction data sets, but if we identify each descriptor $[A, val_A]$ in DISs with an item in transaction data, we can adjust the Apriori algorithm to DISs. For example, we see the object x_1 in Table 1 shows an item set below,

$$ItemSet(x_1) = \{[doors, 2], [persons, 2], [maintenance, poor], [acceptability, poor]\},$$
$$Set\_ItemSet(\psi) = \{ItemSet(x_1), ItemSet(x_2), \cdots, ItemSet(x_6)\}.$$

We term the Apriori algorithm handling the above data structure the DIS-Apriori algorithm, which has the following properties.

(Property 1) The amount of elements in each $ItemSet(x_i)$ is equal to the amount of the attributes.
(Property 2) The decision attribute Dec is usually fixed, and the decision part is $[Dec, val]$. In each item set, only one decision part $[Dec, val]$ must exist.
(Property 3) The DIS-Apriori algorithm is almost the same as the Apriori algorithm for the transaction data set except (Property 1) and (Property 2).

Now, I would like to state my personal opinions on rough set-based rule generation.
(RS-1) In rough sets, at the beginning we need to specify a descriptor $[Dec, val]$. Then, a target set X is defined by $[Dec, val]$, and each subset (defined by condition attributes) of X is examined for rule generation. Thus, the obtained rules characterize the target set X and $[Dec, val]$.
(RS-2) The concept of approximation of a target set X is not only for rule generation but also for general approximation theory on data analysis. The concept of rough sets seems to introduce new research methodology, and there exist several extended researches on approximations [9, 10]. Therefore, it will be better to distinguish researches on extending the concept of approximations from researches on realizing effective rule generators.
(RS-3) For an implication τ and its redundant implication τ', the *support* value preserves monotonicity, i.e., $support(\tau') \leq support(\tau)$, but the *accuracy* value does not preserve monotonicity. Since the criterion $accuracy(\tau)$=1 is usually employed as a first priority in rough sets, the property of monotonicity on the *support* value seems not to be employed.
(RS-4) The logical concepts of soundness (the obtained implication satisfies the constraint of rule) and completeness (any implication satisfying the constraint of rule is obtained) seem to be one factor for ensuring the validity of a rule generation algorithm. In rough sets, Proposition 1 and lower approximation LOW of a target set X seem to ensure the validity of rule generation. However, Skowron and Rauszer proved that to find all minimal reducts is NP-hard [9]. By using the discernibility function, the problem to obtain all lower approximations is translated to SAT problem [9]. According to this result, to obtain all rules is not easy, and completeness of rule generation algorithm seems not have been treated as a topic. Even though it will be difficult to consider completeness, it will be important to study this issue. Otherwise, we may be afraid that there may exist missing rules besides the obtained rules.

Now, I would like to state my personal opinions on Apriori-based rule generation.
(AP-1) Any target set X is not specified, and obtained rules satisfy $support(\tau) \geq \alpha$ and $accuracy(\tau) \geq \beta$ for the specified $\alpha,\ \beta \in [0, 1]$.
(AP-2) In Apriori-based rule generation, the decision attribute is generally not defined, but we can consider the Apriori algorithm with a target set [8] like in rough sets. This is a combination of rough sets and the Apriori algorithm.
(AP-3) The Apriori algorithm is sound and complete for rules, if descriptors are clearly defined. Namely each implication τ obtained by this algorithm satisfies (i) and (ii): (i) $support(\tau) \geq \alpha$ and $accuracy(\tau) \geq \beta$ (soundness), (ii) any implication τ satisfying $support(\tau) \geq \alpha$ and $accuracy(\tau) \geq \beta$ is obtained (completeness). The validity of the NIS-Apriori algorithm is theoretically ensured by these two logical properties. In rule generation and data mining, the system preserving such logical properties is rare.

(AP-4) In order to preserve completeness of rule generation, it is necessary to examine every implication. Here, redundancy of implications and monotonicity of the $support(\tau)$ value can be effectively employed [7, 8]. However, the Apriori algorithm may not be effective to obtain rules specified by the lower α value. If we employ lower α, the Apriori algorithm needs to examine almost all implications.

I indicated my personal opinions on two types of rule generation. I would like to state my personal opinions again for effective rule generators.
(1) In rough set-based rule generation, some rules specified by a descriptor $[Dec, val]$ are obtained. This seems to mean 'local rule generation' from the table data set. In Apriori-based rule generation, any rule specified by α and β is obtained. This seems to mean 'global rule generation' from the table data set.
(2) In the logical properties on soundness and completeness, Apriori-based rule generation seems to be considerable, because the definition of rule is clear. In rough sets, such logical properties seem not to be treated as a topic. The biggest factor for this seems to be related to Skowron and Rauszer's result [9].
(3) In rule generation, Apriori-based rule generation seems to be effective for rules specified by higher α, but rough set-based rule generation (it also makes use of the equivalence classes) seems to be effective for rules specified by lower α. (Actually, the *support* constraint seems not to be employed explicitly.)
(4) I believe that rough set-based rule generation will be suitable from small size data sets, and Apriori-based rule generation specified by higher α will be suitable from large size data sets. The Apriori algorithm with a target set is one combination of two rule generation techniques. It is necessary to combine the advantage of each rule generation technique.

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Interactive Granular Computing (IGrC):
Rough Sets based on Judgment over Dynamic Networks of Complex Granules

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Rough sets introduced by Zdzisław Pawlak[1] play a crucial role in the development of Granular Computing (GrC)[2]. The extension of GrC to Interactive Granular Computing (IGrC) (initiated by Skowron and co-workers[3]), requires generalization of the basic concepts of rough sets and GrC such as *granules* to *complex granules* (including both physical and abstract parts), information (decision) systems to interactive information (decision) systems as well as methods of inducing hierarchical structures of information (decision) systems to methods of inducing hierarchical structures of interactive information (decision) systems. IGrC not only takes into account the granularity of information as used by humans in problem solving, but also interactions with (and within) the real physical world are of the great importance for IGrC. The computations in this IGrC model are realized on the interactive complex granules and that *must* be based on the consequences of the interactions occurring in the physical world. It is worthwhile to cite here the following opinion[4]:

> *It seems that we have no choice but to recognize the dependence of our mathematical knowledge (...) on physics, and that being so, it is time to abandon the classical view of computation as a purely logical notion independent of that of computation as a physical process.*

Consequently, the computational models in IGrC related to the complex phenomena cannot be constructed solely in an abstract mathematical space. They must also take into account continuous interactions with and within the real physical space. In particular, the computational models cannot ignore the laws of physics.

With the interaction rules learned from the acquired data, computations can approximate complex vague concepts related to the expectations of the complex granules (e.g., agents).

The objective of IGrC is also in line with the proposition of Fredrick Brooks (a recipient of the Turing Award). According to him[5]:

> *Mathematics and the physical sciences made great strides for three centuries by constructing simplified models of complex phenomena, deriving, properties from the models, and verifying those properties experimentally. This worked because the complexities ignored in the models were not the essential properties of the phenomena. It does not work when the complexities are the essence.*

The IGrC models, in the form of complex networks of complex granules, have to be created adaptively and autonomously through a process of continuous interaction with reality. On the one hand, due to uncertainty in perception of situations the discovered different local models can be inconsistent with each other but on the other hand their relevant aggregation should lead to the discovery of new knowledge about the perceived situation. It should be noted that models created in the abstract space must be also able to adapt to the changes perceived in the external physical reality.

The main aim of the current research in IGrC is to develop the IGrC models over complex granules. More compound granules are represented by networks of interacting simpler granules changing in time. Any IGrC model must also be able to direct the attention of complex granules (e.g., agents) to focus on the significant fragments of reality that are measured by the sensors and explored by the actuators used in performing the actions or plans. Results of interactions are collected in information systems (data tables), which constitute fragments of the complex granules. Following

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another Turing Award winner, Leslie Valiant, these tables are then aggregated to create new complex granules as computational building blocks for cognition[6].

There are many challenges related to IGrC. Some of them are related to reasoning, called *adaptive judgment*[7], about properties of complex granules and interactive computations over them. One of the main aim of *adaptive judgment* performed by complex granules (e.g., agents) is to derive conclusions regarding selection of action(s) which should be currently initiated (or terminated). The actions are activated on the basis of satisfiability of some complex vague concepts labelled by actions. It should be noted that these concepts may change. Adaptive learning of such concepts based on judgment is a grand challenge[8]. The whole process towards inducing approximation of these vague concepts labelled by actions, which are initiated on the basis of satisfiability of these concepts, may be treated as a process of discovery of a *complex game*. In such a game the concepts (together with assigned relevant judgment mechanisms to them) can be treated as players who by using their judgment mechanisms are deriving arguments *for* and *against* the satisfiability of these concepts on the basis of information about the perceived situation. Next, there are other judgment mechanisms, in the hands of a judge, that can be used to resolve conflicts among the collected arguments to select the winning player (concept). Then action-labelling the winning concept is initiated.

It should be also noted that approximation of the complex vague concept should be based on adaptive judgment rather than on partial inclusion of sets only which is widely used in the rough set approach. The former approach is much more general than the latter one. The approach based on judgment is especially relevant when in data analysis it is required to have a deeper judgment about the perceived complex situation related to classification of complex vague concepts. The approach based on partial containment of sets only is not satisfactory for dealing with many real-life applications, where more advanced judgment should be made to identify the perceived situation, to classify it relative to the complex vague concepts or to reason about risk for supporting the decision making. In particular, there is a need for developing new logical tools for reasoning based on judgment toward approximation of complex vague concepts and to the rough set approach based on adaptive judgment performed over computations on complex granules. This, in particular, creates a room for extensions of rough sets to adaptive rough sets and rough sets over distributed networks of granules changing in time.

Another challenging research direction is related to self-organization in synthesis of complex granules and their networks.

Finally, it is worthwhile mentioning that IGrC is also in agreement with the recently raised discussions about the Turing test for intelligence. In addition to linguistic aspects and reasoning, it incorporates perception and actions, and it follows what Leslie Valiant's calls *ecorithms*[9].

The proposed model of computation based on complex granules seems to be of fundamental importance to developing of intelligent systems dealing with complex phenomena, in particular in such areas as Data Science, Internet of Things, Wisdom Web of Things, Cyber-Physical Systems, Complex Adaptive Systems, Natural Computing, Software Engineering, applications based on Blockchain Technology, etc[10].

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[6] see https://people.seas.harvard.edu/~valiant/researchinterests.htm

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Three topics on rough set research

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Abstract

In past ten plus years, my brain has been pretty much occupied with the magic number **three**. It forced me to think in threes, to view the world in threes, and to organize myself in threes, where the threes can be three parts, three components, three perspectives, three levels, three stages, three questions, three tasks, and many more. I introduced a theory of three-way decision to focus on the philosophy, theory, and practice of thinking and processing in threes. In this short note, inevitably, I present a very personal view of the past, present, and future rough set research by looking at three topics.

1. The two sides of rough set theory: The importance of semantics

Conceptual and computational formulations are well discussed topics in many disciplines, for example, mathematics, statistics, physics, chemistry, and many more. A conceptual formulation emphasizes on the meaning and interpretation of the concepts and notions of a theory, whereas a computational formulation focuses on procedures and algorithms for constructing these notions. The two formulations are the two sides of the same coin; it is essential to pay equal attention to both.

In a recent paper [11], I examined conceptual and computational formulations of rough sets. Except for a few earlier studies (for example, see [4] and [6]), computational formulations dominate research in rough sets in the past and the present. An oversight of conceptual formulations makes an in-depth understanding of rough set theory very difficult, which creates hurdles

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for further development of rough set theory in the future. I plea for more attention to the semantics of rough set theory.

2. Three-way decision: A step further

Approximating an undefinable set/concept by definable sets/concepts is one of the basic ingredients of rough set theory [7]. There are two mathematically equivalent definitions of approximations that are represented in different forms. One is a pair of lower and upper approximations and the other is three pair-wise disjoint positive, boundary, and negative regions. It is important to point out that we may start with any one of the two and define the other by using the former, although Pawlak defined the three regions by using the pair of approximations.

The formulation based on a pair of approximations enables us to connect rough sets to modal logic, interval sets [9], orthopairs [1], fuzzy sets (in terms of two alpha-cuts), shadowed sets [8], and many others. The formulation based on three regions motivates the introduction of a theory of three-way decision [10]. There is a growing interest in research on three-way decision [2, 3, 5, 12, 13, 15]. The interplay of rough sets and three-way decision may provide good opportunities for us to move a step further in advancing the two theories.

3. Triangulation: Making connections

There have been extensive studies on integrating rough set theory and another theory, for example, fuzzy sets, granular computing, data mining, machine learning, formal concept analysis, and so on. The combinations of two theories are interesting. In the light of three-way decision as thinking in threes, it might be the time to look at possibilities of integrating three theories in which rough set theory is one of them.

A triangulation of three theories offers a middle ground that avoids the simplicity of combining two theories on one hand and the complexity of four or more theories on the other. With three theories, we have three individual theories, three pairs of two theories, and one unification of the three theories [13]. In this research trend of triangulation, we have already witnessed some initiatives, for example, “formal concept analysis, rough sets, and granular computing” [14], “granular computing, shadowed sets, and three-way decisions”[1], and “formal concept analysis, rough sets, and three-way decisions”[2]. Connected we are stronger: Triangulation of three theories provides a wider context for rough set research, expands rough set applications, and engages researchers from other communities.

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Some Thoughts About the Future and Role of Rough Set Paradigm

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In my view, the rough set theory, as introduced by Professor Zdzislaw Pawlak, is of such a fundamental nature that, in the current and future research, it has the potential to play a role similar to the set theory in mathematics. The mathematical set theory underlies other branches of mathematics, such as algebra, mathematical analysis, calculus etc., branches which are extensively used in numerous applications in Science (including Computer and Data Science), Engineering, Economics, Statistics, just to mention a few. In those application areas, the set theory is most often not used directly, but without it, the useful methodologies and algorithms of those application areas would not exist.

In line with the above analogy, I think that rough set theory is, first of all, the basic building platform, enhancing our understanding of the issues involved, based on which application area-oriented theories, methodologies and algorithms could be developed. In some areas, the rough set theory is directly applicable, but in many of them is not, for example, due to noise in measured data or to high degree of its randomness. There are different requirements and data quality is different in data mining or market research applications, versus supervised learning or sensor-based control algorithm development. This is why there is a need for more application-oriented research, developing theories and methods with the underlying rough set, or its extensions, paradigms to help in solving practical problems existing in diverse application areas.

I would foresee two major directions. First, development of new extensions or theories, with the underlying idea of rough sets, which are more applicable to real-world problems than the rough set theory itself. Second, working on specific practical applications, in teams with researchers or practitioners from application areas (e.g. engineers, market researchers, doctors, chemists), to help solving their specific problems, while using our expertise in rough sets and its derivative theories. It is essential to work jointly with an application domain experts to solve real, not invented, practical problems and to take advantage of the combined rough set and application domain expertise.

This may require stronger involvement of rough set researchers in industrial projects, and generally, shifting the research focus from theoretical to more practical. As far as I can see, there is a shortage of application-oriented papers and projects involving rough set methodology. Some other areas, such as neural nets (or deep learning, as they are called these days), have gained prominence and popularity, and consequently very significant research funding, by relentless push for practical applications. There is no reason why it should not be the case for rough set-based approaches and methodologies.

The other aspect is availability of software to support rough set-oriented research. Particularly, when it comes to applications, it is rather frustrating to start from scratch, trying to tediously program existing rough set methods to experiment with application problems, when the focus should be on an application. The easy availability, and persistent development, of a standard suite of programs to support application-oriented rough set research, would be extremely helpful to application researchers and would go long way towards popularizing the rough set paradigms. Perhaps an “Open Rough Set Software Foundation” could be initiated, to involve numerous programmers worldwide in rough set software development and make the software, including the source code, available for free to researchers and businesses? I believe that the key to future progress and popularity of rough set-based approaches is the wide-spread application-oriented research in Academia and Industry, with confirmed and deployed successful applications.