INTRODUCTION TO ROUGH SETS

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AGENDA

ROUGH SETS (RS): BASIC CONCEPTS

GENERALIZATIONS OF RS

RELATIONSHIPS OF RS WITH OTHER APPROACHES

RS AND (APPROXIMATE) BOOLEAN REASONING

RS & GRANULAR COMPUTING:
APPROXIMATION OF (COMPLEX) VAGUE CONCEPTS

WHAT NEXT?
RS AND INTERACTIVE COMPUTATIONS ON COMPLEX GRANULES

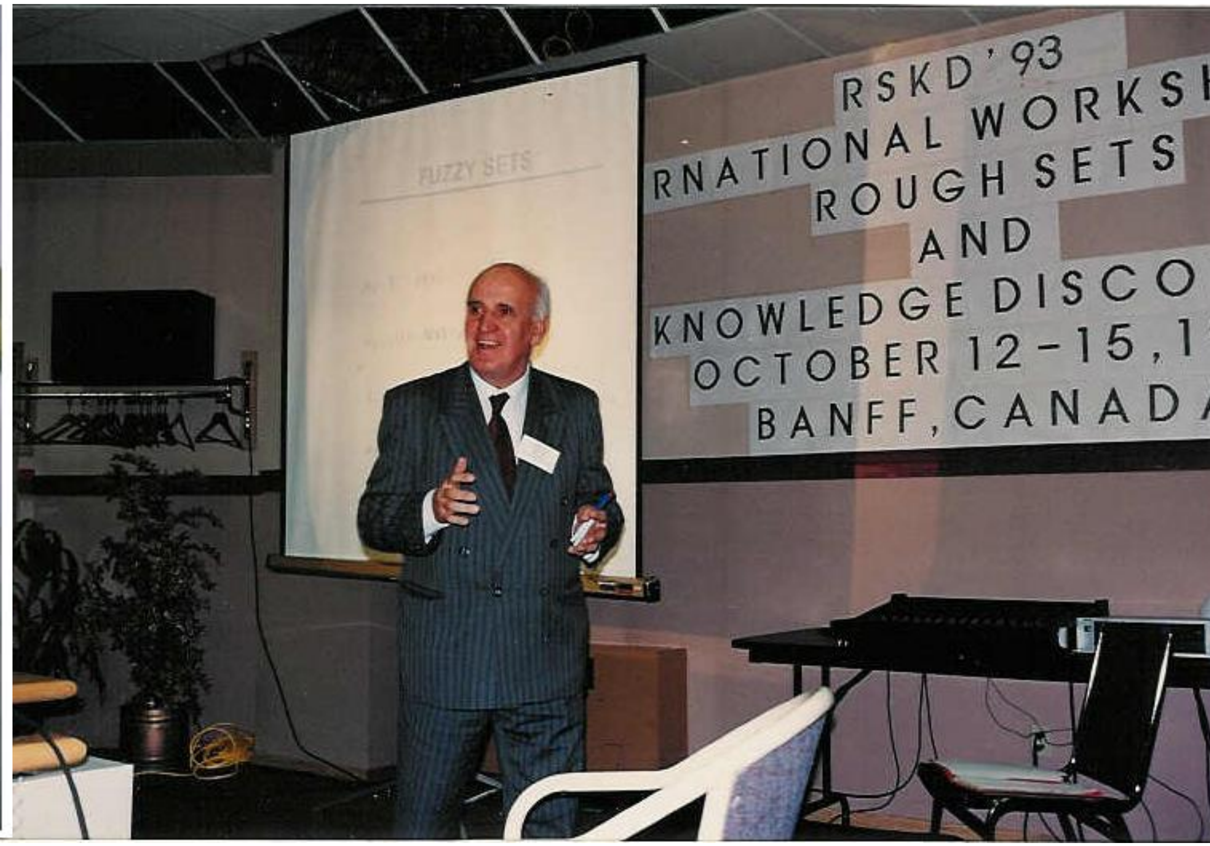
BASIC CONCEPTS OF ROUGH SETS

- Information/Decision Systems (Data Tables)
- Indiscernibility and Discernibility
- Set Approximation
- Reducts and Core
- Rough Membership
- Dependency of Attributes
- Decision Rules

RUDIMENTS OF ROUGH SETS

Pawlak, Z.: Rough sets. International Journal of Computer and Information Sciences 11 (1982)

Pawlak, Z.: Rough sets. Theoretical Aspects of Reasoning About Data. Kluwer (1991)



Now thousands of papers http://rsds.univ.rzeszow.pl/

INFORMATION SYSTEMS

	Age	LEMS
x1	16-30	50
x2	16-30	0
x3	31-45	1-25
x4	31-45	1-25
x5	46-60	26-49
x6	16-30	26-49
x7	46-60	26-49

- *IS* is a pair (*U*, *A*)
- *U* is a non-empty finite set of objects.
- *A* is a non-empty finite set of attributes such that $a: U \rightarrow V_a$ for every $a \in A$.
- V_a is called the value set of *a*.

DECISION SYSTEMS

$$DT = (U, A, d) \quad d \notin A$$

condition attributes

decision attribute

$$d : U \to V_d$$

	Age	LEMS	Walk
x1	16-30	50	yes
x2	16-30	0	no
x3	31-45	1-25	no
x4	31-45	1-25	yes
x5	46-60	26-49	no
x6	16-30	26-49	yes
x7	46-60	26-49	no

decision classes

$$X_i = \{x \in U : d(x) = i\} \;\; for \;\; i \in V_d$$

inconsistent cases

decision systems:

- consistent
- inconsistent

UNCERTAINTY IN OBJECT PERCEPTION
INDISCERNIBILITY RELATIONS

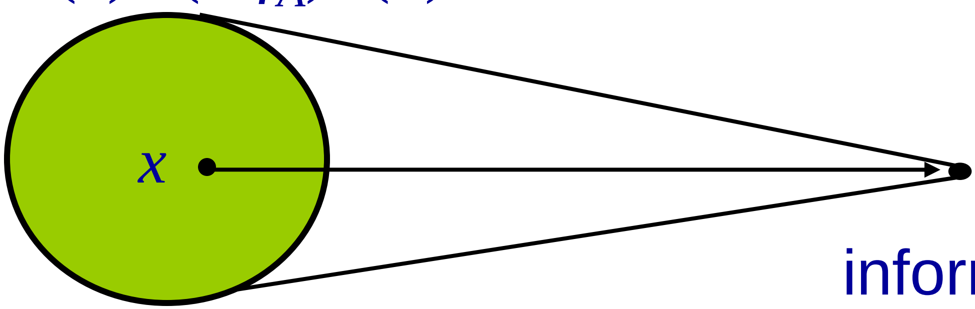
information system (data table)

$$IS = (U, A)$$

$$U = \{x_1, \ldots, x_n\},\ A = \{a_1, \ldots, a_m\}$$

	a_1	a_2	…	a_m
x_1	v_1	v_2	…	v_m
	…	…	…	…

$N(x)=(Inf_A)^{-1}(u)$



neighborhood of x

$u=Inf_A(x)$

information signature of x

$$x IND(A) y \ \ iff \ \ Inf_A(x) = Inf_A(y)$$

$$\tau$$

tolerance or similarity

$$IND(B) \ \ for \ \ B \subseteq A$$

$$[x]_{IND(B)} = [x]_B = \{y \in U : x IND(B) y\}$$

$$U / B = \{[x]_B : x \in U\}$$

DECISION SYSTEMS

U A d

	Age	LEMS	Walk
x1	16-30	50	yes
x2	16-30	0	no
x3	31-45	1-25	no
x4	31-45	1-25	yes
x5	46-60	26-49	no
x6	16-30	26-49	yes
x7	46-60	26-49	no

$$DT = (U, A, d) \quad d \notin A$$

condition attributes

decision attribute

$$d : U \rightarrow V_d$$

decision classes

$$X_i = \{x \in U : d(x) = i\} \ \ for \ \ i \in V_d$$

inconsistency

Generalized decision:

$$\partial_B : U \rightarrow P(V_d) \ where \ B \subseteq A$$

$$\partial_B(x) = \{v' : \exists x' (xIND(B)x' \wedge d(x') = v')\} = d([x]_B)$$

Remark. Possible generalization for many decisions.

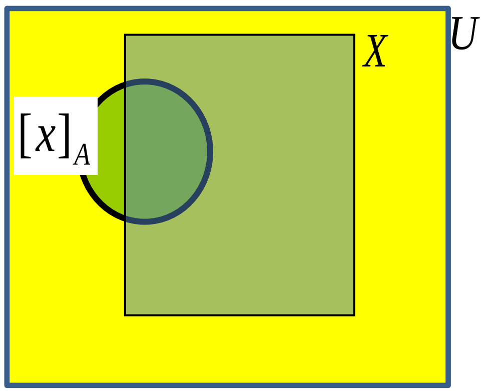
UNCERTAINTY IN OBJECT PERCEPTION
APPROXIMATION OF DECISION CLASSES

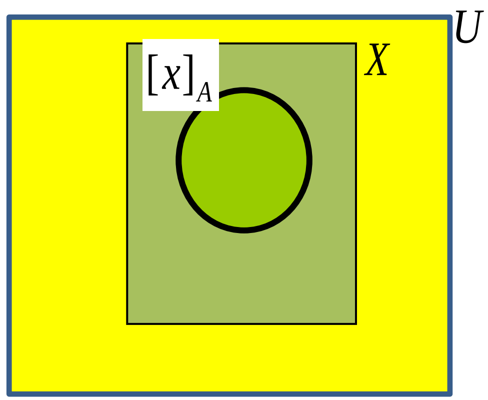
$$X = \{x \in U : d(x) = 1\}$$

decision system (data table)

$$DT = (U, A, d)$$

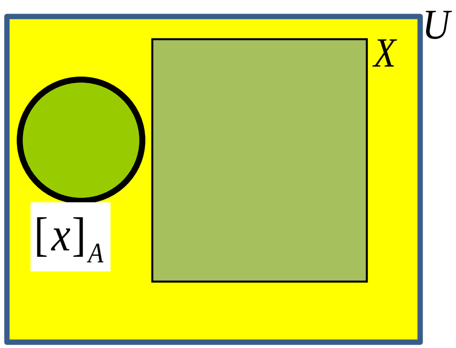
	a_1	a_2	$\dots$	a_m	d
x_1	v_1	v_2	$\dots$	v_m	1
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$





$$[x]_{IND(A)} = [x]_A =$$

$$\{y \in U : xIND(A)y\}$$



A-definable sets: unions of indiscernibility classes

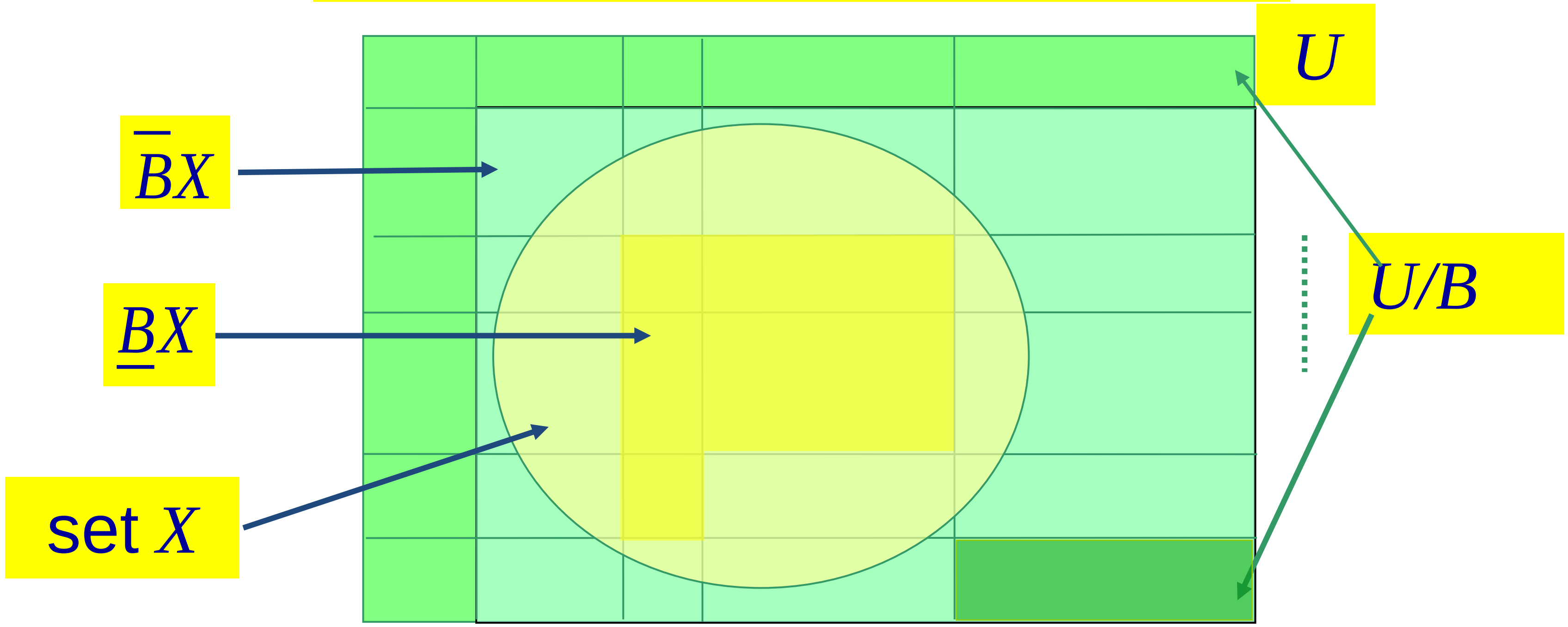
PROBLEM: Is a given decision class definable (relative to A)?

LOWER AND UPPER APROXIMATION

$$X \subseteq U,\ B \subseteq A$$

$$\underline{B}X = \bigcup\{Y \in U/B : Y \subseteq X\}$$

$$\overline{B}X = \bigcup\{Y \in U/B : Y \cap X \neq 0\}$$



BOUNDARY REGION

$$BN_B(X) = \overline{B}X \setminus \underline{B}X$$

ROUGH SETS

BOUNDARY REGION

$$BN_B(X) = \overline{B}X \setminus \underline{B}X$$

CRISP SET

$$BN_B(X) = \varnothing$$

ROUGH SET

$$BN_B(X) \neq \varnothing$$

UNCERTAINTY
IN SIGNATURES OF OBJECTS

- missing values – different interpretations
- uncertainty in attribute value measurement
- noise
- …

DISCERNIBILITY

$$x DIS(B) y \ \ iff \ \ non(x IND(B)) y$$

However, this is only the simplest case!

AN EXAMPLE

	Age	*LEMS*	*Walk*
x1	16-30	50	yes
x2	16-30	0	no
x3	31-45	1-25	no
x4	31-45	1-25	yes
x5	46-60	26-49	no
x6	16-30	26-49	yes
x7	46-60	26-49	no

- Let *W* = {x | *Walk*(x) = yes}.

$$\underline{A}W = \{x1, x6\},$$

$$\overline{A}W = \{x1, x3, x4, x6\},$$

$$BN_A(W) = \{x3, x4\},$$

$$U - \overline{A}W = \{x2, x5, x7\}.$$

- The decision class, *Walk*, is rough since the boundary region is not empty.

LOWER & UPPER APPROXIMATIONS

U	*Headache*	*Temp.*	*Flu*
u1	Yes	Normal	No
u2	Yes	High	Yes
u3	Yes	Very-high	Yes
u4	No	Normal	No
u5	*No*	*High*	*No*
u6	***No***	***Very-high***	***Yes***
u7	*No*	*High*	*Yes*
u8	***No***	***Very-high***	***No***

Elementary sets of indiscernibility relations defined by

$B = \{Headache, Temp.\}$ are $\{u1\}, \{u2\}, \{u3\}, \{u4\}, \{u5, u7\}, \{u6, u8\}$.

$X1$ = Flu(yes) = $\{u2, u3, u6, u7\}$

Lower approximation:

$\underline{B}X1 = \{u2, u3\}$

Upper approximation:

$\overline{B}X1 = \{u2, u3, u6, u7, \mathbf{u8, u5}\}$

$X2$ = Flu(no) = $\{u1, u4, u5, u8\}$

Lower approximation:

$\underline{B}X2 = \{u1, u4\}$

Upper approximation:

$\overline{B}X2 = \{u1, u4, u5, u8\ \mathbf{u7, u6}\}$

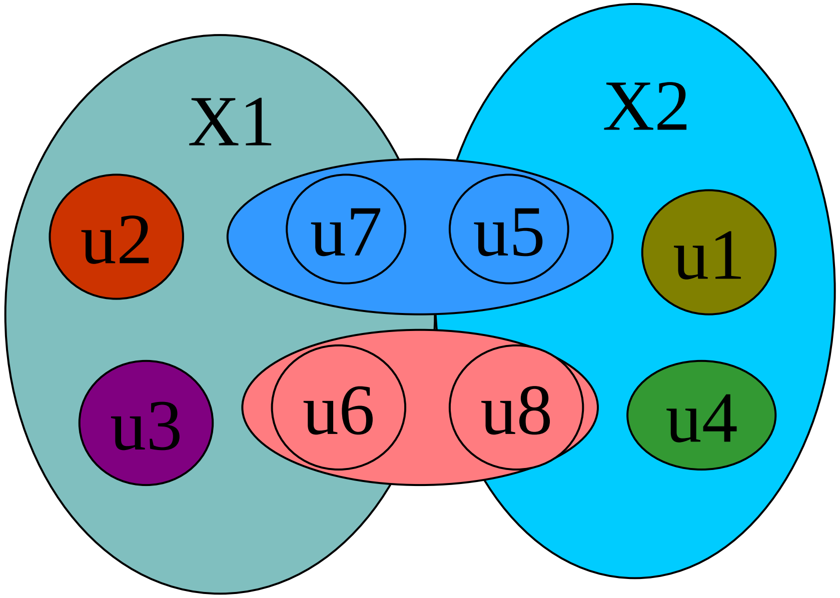
LOWER & UPPER APPROXIMATIONS

$R = \{Headache, Temp.\}$
$U/R = \{\{u1\}, \{u2\}, \{u3\}, \{u4\}, \{u5, u7\}, \{u6, u8\}\}$

$X1$ = Flu(yes) = $\{u2,u3,u6,u7\}$
$X2$ = Flu(no) = $\{u1,u4,u5,u8\}$

$\underline{B}X1 = \{u2, u3\}$
$\overline{B}X1 = \{u2, u3, u6, u7, \boldsymbol{u8, u5}\}$

$\underline{B}X2 = \{u1, u4\}$
$\overline{B}X2 = \{u1, u4, u5, u8, \boldsymbol{u7, u6}\}$



ACCURACY
OF APPROXIMATION

$$\alpha_B(X) = \frac{|\underline{B}(X)|}{|\overline{B}(X)|}$$

where |X| denotes the cardinality of $X \neq \emptyset$.

Obviously $0 \leq \alpha_B \leq 1$.

If $\alpha_B(X) = 1,$ *X* is *crisp* with respect to *B*.

If $\alpha_B(X) < 1,$ *X* is *rough* with respect to *B.*

PROPERTIES OF APPROXIMATIONS

$$\underline{B}(X) \subseteq X \subseteq \overline{B}X$$

$$\underline{B}(\phi) = \overline{B}(\phi) = \phi, \underline{B}(U) = \overline{B}(U) = U$$

$$\overline{B}(X \cup Y) = \overline{B}(X) \bigcup \overline{B}(Y)$$

$$\underline{B}(X \cap Y) = \underline{B}(X) \cap \underline{B}(Y)$$

$$X \subseteq Y \text{ implies } \underline{B}(X) \subseteq \underline{B}(Y) \text{ and } \overline{B}(X) \subseteq \overline{B}(Y)$$

PROPERTIES OF APPROXIMATIONS

$$\boxed{\begin{array}{l} \underline{B}(X \cup Y) \supseteq \underline{B}(X) \cup \underline{B}(Y) \\ \overline{B}(X \cap Y) \subseteq \overline{B}(X) \bigcap \overline{B}(Y) \end{array}}$$

$$\underline{B}(-X) = -\overline{B}(X)$$

$$\overline{B}(-X) = -\underline{B}(X)$$

$$\underline{B}(\underline{B}(X)) = \overline{B}(\underline{B}(X)) = \underline{B}(X)$$

$$\overline{B}(\overline{B}(X)) = \underline{B}(\overline{B}(X)) = \overline{B}(X)$$

where -*X* denotes *U* \ *X*.

POSITIVE REGION OF DECISION SYSTEM

DT = (*U*,*A*,*d*)

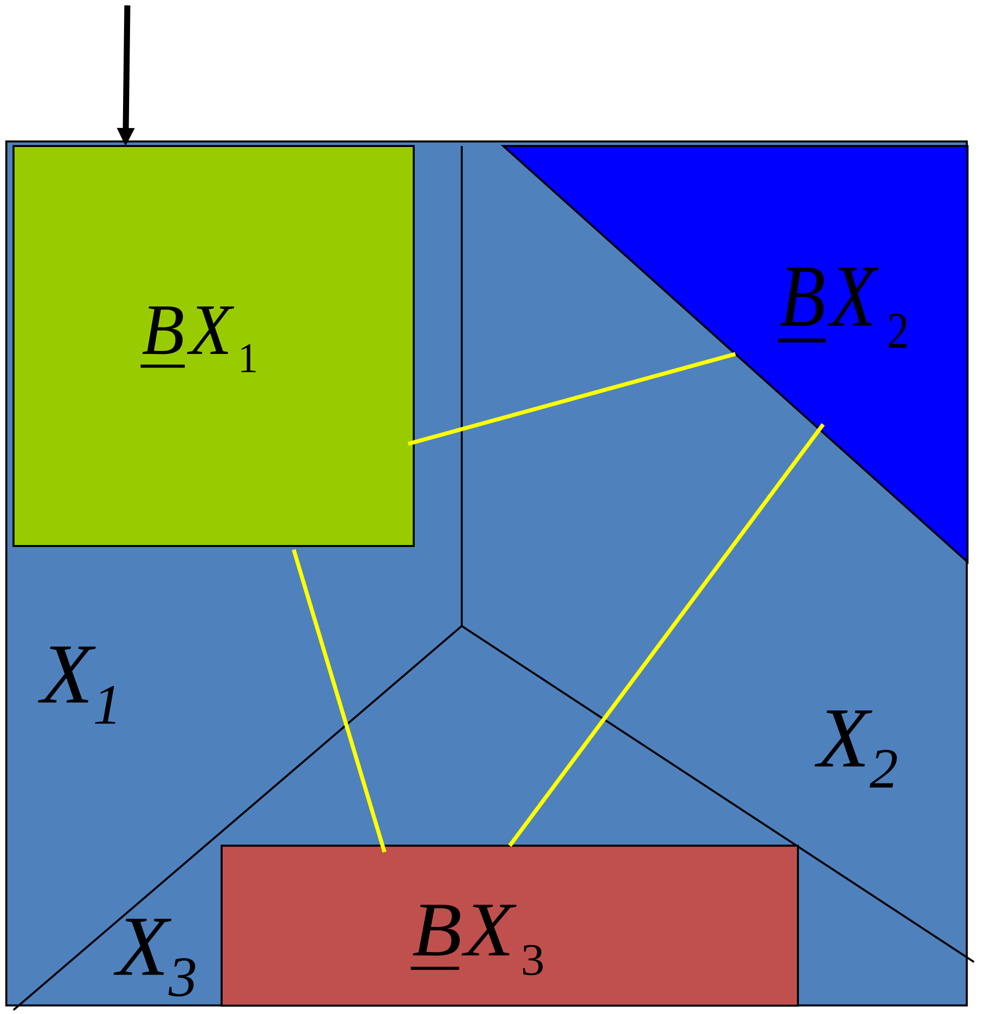
For $B \subseteq A$ we define B-positive region of d:

$$POS_B(d) = \bigcup_{X \in U/d} \underline{B}X$$

POSITIVE REGION OF DECISION SYSTEM (*U,A,d*)

Decision classes:

$U/d=\{X_1,X_2,X_3\}$



DEPENDENCIES IN *DT=(U,A,d)*

$B \subseteq A$. *d* depends on *B* in degree *k* $(0 \leq k \leq 1)$,

$B \Rightarrow_k d$, if

$$k = \gamma(B,d) = \frac{|POS_B(d)|}{|U|}$$

DATA REDUCTION

MINIMAL SETS OF CONDITION ATTRIBUTES PRESERVING DISCERNIBILITY CONSTRAINTS: REDUCTS

- between discernible objects in a given information system ➔ **reducts in information systems**
- between objects from different decision classes ➔ **decision reducts**
- between a given object x with a decision *i* and other objects with a decision different from *i* ➔ **local reducts relative to the object** x
- ...

REDUCTS IN INFORMATION SYSTEMS

- For a given information system $IS=(U, A)$ we are searching for minimal subsets $B \subseteq A$ such that

$$IND(B) = IND(A)$$

- *RED(IS)* or *RED(A)* – the set of all reducts in *IS*.
- $CORE(IS)=\bigcap RED(IS)$.

DECISION REDUCTS IN *DT=(U,A,d)*

- $B \subseteq A$ is called a *decision reduct* of *DT*, if *B is a minimal subset of A such that*

$$POS_B(d) = POS_A(d).$$

- $RED(DT)$ is the set of all *decision reducts* of *DT.*
- $CORE(DT) = \bigcap RED(DT).$
- Another constraint for decision reducts:

$$\partial_A = \partial_B.$$

LOCAL REDUCTS

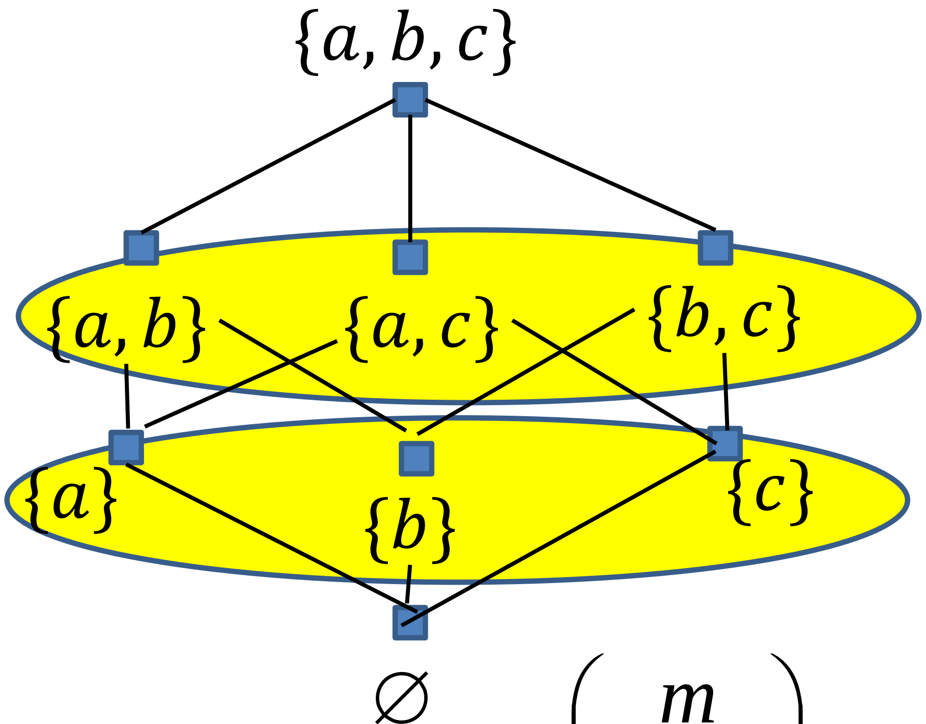
$$DT = (U, A, d) \ and \ x \in U$$

A **local reduct relative to a given object** x minimal susbset $B_x \subseteq A$ preserving discernibility of x with all objects y discernible from x, i.e., such that

$$\partial_A(x) \neq \partial_A(y).$$

PROBLEMS WITH REDUCTS

The set of reducts of any *IS* =(*U*,*A*) in a lattice ($P(A)$,$\subseteq$) of subsets of attributes creates an **antichain** (with inclusion as a partial order)



$$\binom{m}{\left\lfloor m/2 \right\rfloor} \quad where\ m = |A|$$

PROBLEMS WITH REDUCTS

- The number of reducts can be large, e.g., some information systems can have exponential number of reducts with respect to the number of attributes

- Problems of computing minimal reducts are of high complexity (NP-hard).

Fortunately, different efficient heuristics for computing relevant reducts or sets of reducts, e.g., based on BOOLEAN REASONING were developed.

DECISION RULES

FORMULAS
OVER DECISION SYSTEM

$$DT = (U, A, d),\ B \subseteq A \cup \{d\}$$

$F(B)$: the smallest set

SYNTAX 1. consisting of descriptors:

$$a = v \ \ for \ \ a \in B, v \in V_a$$

2. closed with respect to $\wedge, \vee, \neg$

SEMANTICS

$$\|a = v\|_{DT} = \{x \in U : a(x) = v\}$$

$$\|\alpha \wedge \beta\|_{DT} = \|\alpha\|_{DT} \cap \|\beta\|_{DT}$$

$$\|\alpha \vee \beta\|_{DT} = \|\alpha\|_{DT} \cup \|\beta\|_{DT}$$

$$\|\neg\alpha\|_{DT} = U - \|\alpha\|_{DT}$$

DESCISION RULES

$$DT = (U, A, d) - \text{decision system}$$

$$\text{Decision rule}$$

$$a_{i_1} = v_{i_1} \wedge \ldots \wedge a_{i_k} = v_{i_k} \Rightarrow d = v \in V_d$$

$$\text{Generalized decision rule}$$

$$a_{i_1} = v_{i_1} \wedge \ldots \wedge a_{i_k} = v_{i_k} \Rightarrow \partial_A = V \subseteq V_d$$

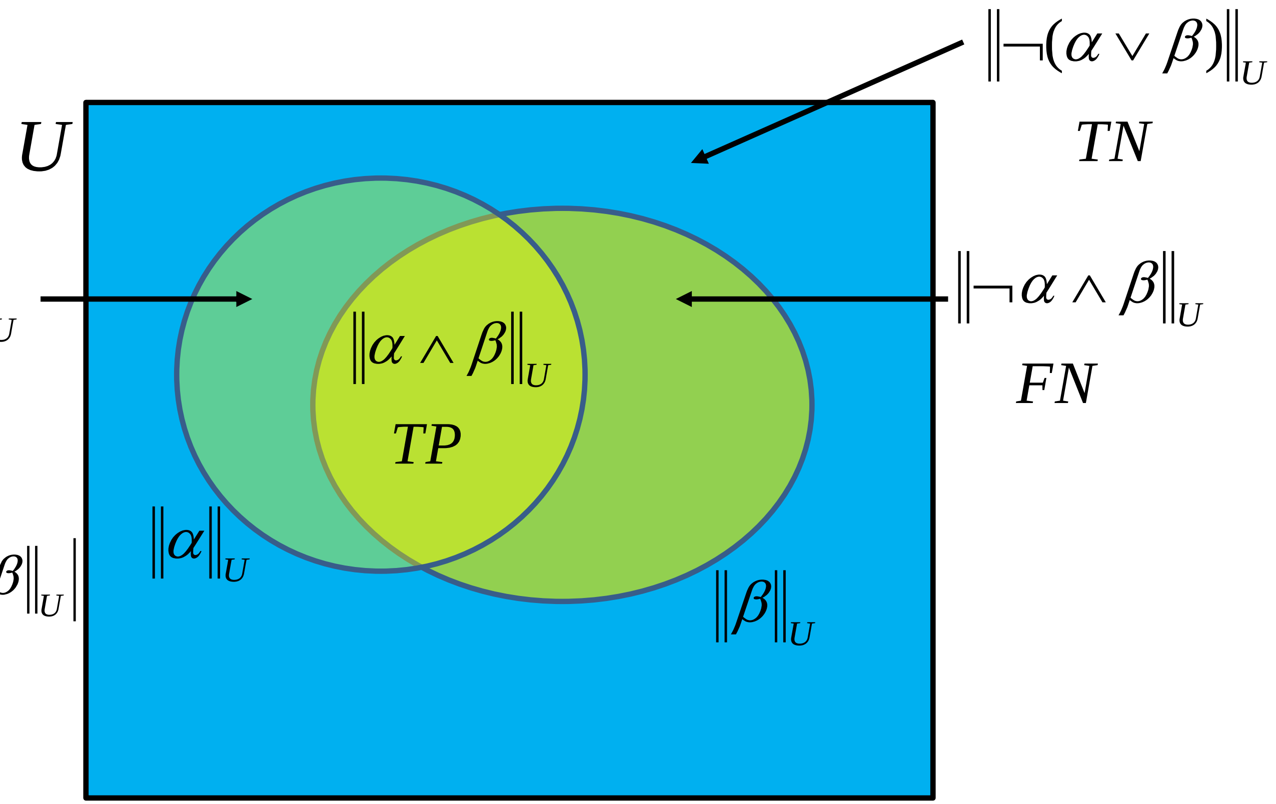
DESCISION RULES

Decision rule is true in DT iff

$$\left\| a_{i_1} = v_{i_1} \wedge \ldots \wedge a_{i_k} = v_{i_k} \right\|_{DT} \subseteq \left\| \partial_A = V \right\|_{DT}$$

In case when the inclusion is not crisp the partial truth can be considered.

ROUGH INCLUSION



$$\mathrm{supp}_U(\alpha, \beta) = \left|\|\alpha \wedge \beta\|_U\right|$$

$$\mathrm{conf}_U(\alpha, \beta) = \frac{\left|\|\alpha \wedge \beta\|_U\right|}{\left|\|\alpha\|_U\right|} (precision) \quad \mathrm{cov}_U(\alpha, \beta) = \frac{\left|\|\alpha \wedge \beta\|_U\right|}{\left|\|\beta\|_U\right|} \ (TPR,\ sensitivity,\ recall)$$

$$specificity_U(\alpha, \beta) = \frac{\left|\|\neg(\alpha \vee \beta)\|_U\right|}{\left|\|\neg\beta\|_U\right|} \ (TNR)$$

$$FPR = 1 - specificity$$

Jan Łukasiewicz (1913)

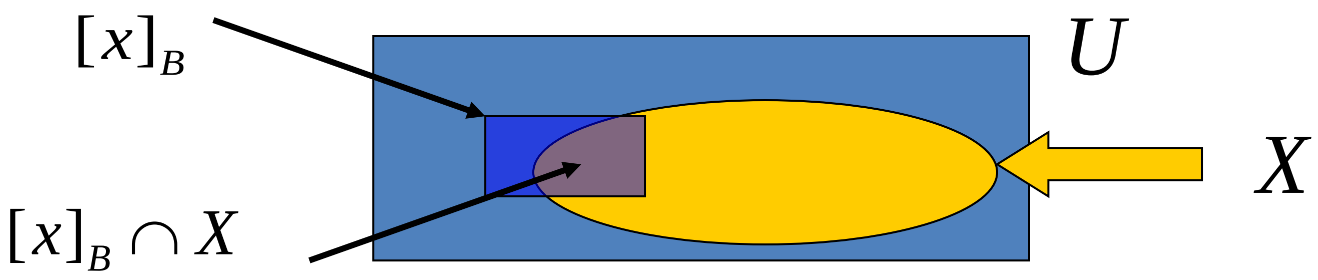
ROUGH MEMBERSHIP

- The rough membership function quantifies the degree of relative overlap between the set *X* and the equivalence class $[x]_B$ to which *x* belongs.

$$\mu_X^B : U \to [0,1] \qquad \mu_X^B(x) = \frac{|[x]_B \cap X|}{|[x]_B|}$$

- The rough membership function can be interpreted as a frequency-based estimate of $P(x \in X \mid u)$,

  where $u=[x]_B$ is the equivalence class of *IND(B)* to which *x* belongs .



ROUGH MEMBERSHIP

$$\mu_X^B(x) = 1 \ \ iff \ \ [x]_B \subseteq X$$

$$\mu_X^B(x) = 0 \ \ iff \ \ [x]_B \subseteq U \setminus X$$

$$\mu_{U \setminus X}^B(x) = 1 - \mu_X^B(x)$$

$$\mu_{X \cap Y}^B(x) \leq \min(\mu_X^B(x), \mu_Y^B(x))$$

$$\mu_{X \cup Y}^B(x) \geq \max(\mu_X^B(x), \mu_Y^B(x))$$

$$\underline{B}X = \{x \in U : \mu_X^B(x) = 1\}$$

$$\overline{B}X = \{x \in U : \mu_X^B(x) > 0\}$$

RELATIONSHIPS OF RS WITH OTHER APPROACHES

- **Vague concepts in philosophy**
- Fuzzy sets
- **Dempster-Shafer theory**
- **Boolean reasoning**
- Statistics
- Logics and algebras
- Formal concept analysis
- Mereology
- Mathematical morphology
- ...

VARIABLE PRECISION ROUGH SET MODEL (VPRS)

- The formulae for the lower and upper approximations can be generalized to some arbitrary level of precision $\pi \in (0.5, 1]$ by means of the rough membership function

$$\underline{B}_{\pi} X = \{x \mid \mu_X^B(x) \geq \pi\}$$

$$\overline{B}_{\pi} X = \{x \mid \mu_X^B(x) > 1 - \pi\}.$$

- Note: the lower and upper approximations as originally formulated are obtained as a special case with $\pi = 1.$

DEMPSTER-SHAFER THEORY
(evidence theory)

$$\Theta - \textit{frame of discernment (set of decisions)}$$

$$m : P(\Theta) \rightarrow [0,1] \ \textit{mass function}$$

$$m(\emptyset) = 0$$

$$\sum_{\Delta \subseteq \Theta} m(\Delta) = 1$$

$$\boxed{\begin{array}{l} Bel(\Delta) = \sum_{\Gamma \subseteq \Delta} m(\Gamma) \ \textit{belief function} \\ Pl(\Delta) = \sum_{\Gamma \cap \Delta \neq \emptyset} m(\Gamma) \ \textit{plausibility function} \end{array}}$$

G. Shafer, Mathematical theory of evidence, Princeton University Press, 1976

RS & i DEMPSTER-SHAFER THEORY

dec. system: DT=(U,A,d),

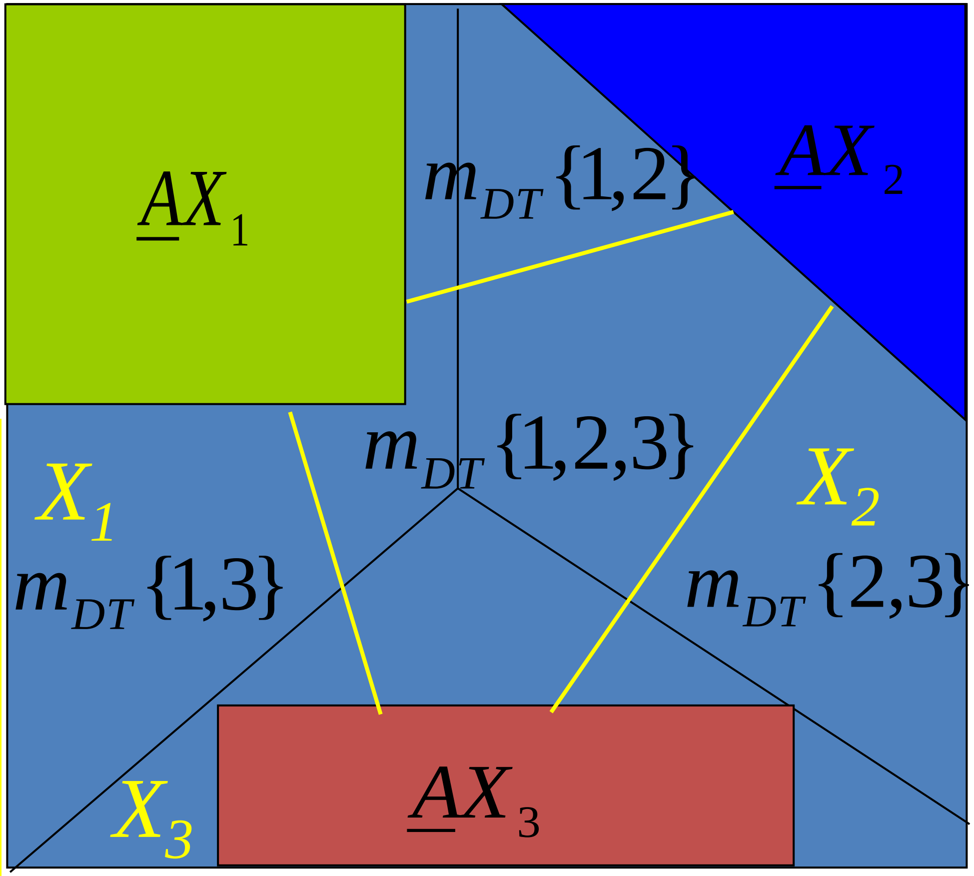
gen.decision: $\delta_A(x) = d([x]_A)$

$$m_{DT}(\Delta) = \frac{\left|\{x \in U : \partial_A(x) = \Delta\}\right|}{|U|}$$

$$\Delta \subseteq \{1,2,3\}$$

$$Bel_{DT}\{1,2\} = \sum_{\Gamma \subseteq \{1,2\}} m_{DT}(\Gamma) = \frac{\left|\underline{A}(X_1 \cup X_2)\right|}{|U|}$$

$$Pl_{DT}\{1,2\} = \sum_{\Gamma \cap \{1,2\} \neq \emptyset} m_{DT}(\Gamma) = \frac{\left|\overline{A}(X_1 \cup X_2)\right|}{|U|}$$



DEMPSTER-SHAFER THEORY
rule of combination

$$m_1 \otimes m_2(\emptyset) = 0$$

$$m_1 \otimes m_2(\Delta) = \frac{\sum_{A \cap B = \Delta} m_1(A) m_2(B)}{1 - \sum_{A \cap B = \emptyset} m_1(A) m_2(B)} \quad for \;\; \emptyset \neq \Delta \subseteq \Theta$$

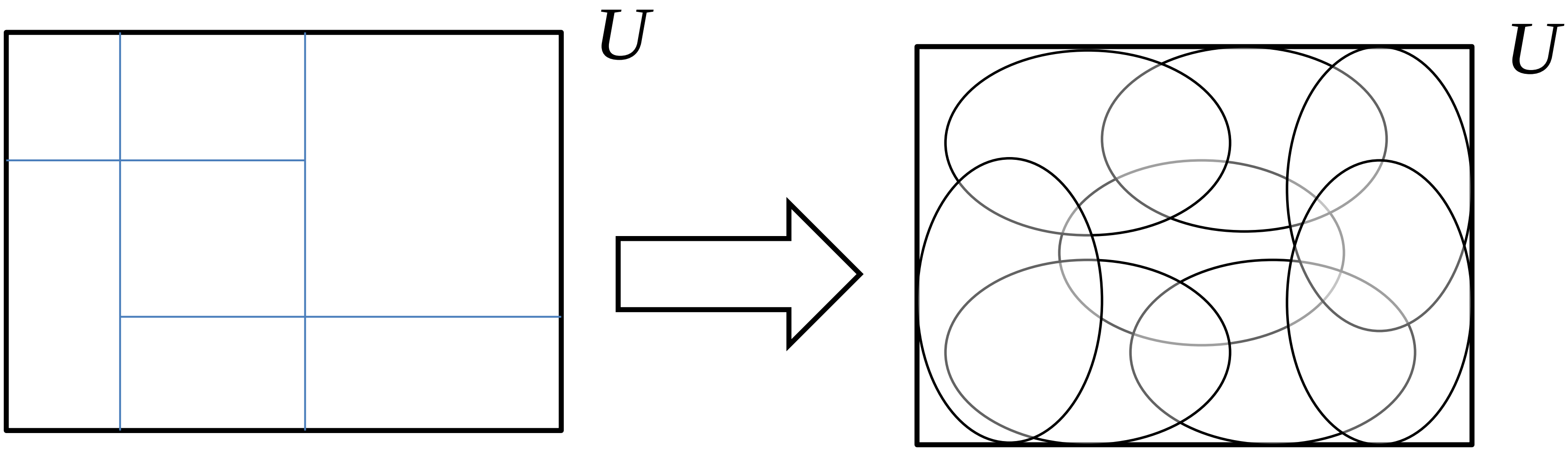
How to define operation $\circ$ *on decision tables*

such that

$$m_{DT_1} \otimes m_{DT_2} = m_{DT_1 \circ DT_2} \; ?$$

GENERALIZATIONS OF ROUGH SETS

FROM PARTITIONS TO COVERINGS



$$\Im - covering:$$

$$\Im \subseteq P(U) \ and \ \cup \Im = U$$

Algorithmic issues:
- discovery of relevant coverings
- relevant family of definable sets
- searching for relevant approximation spaces and operations
-…

GENERALIZATIONS OF ROUGH SETS
e.g., based on TOLERANCE OR SIMILARITY

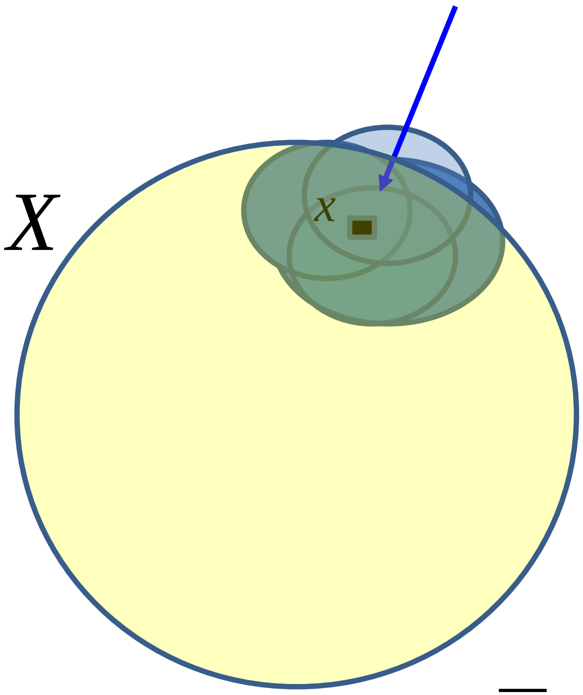
$$\tau_a \subseteq V_a \times V_a - tolerance\ or\ similarity$$

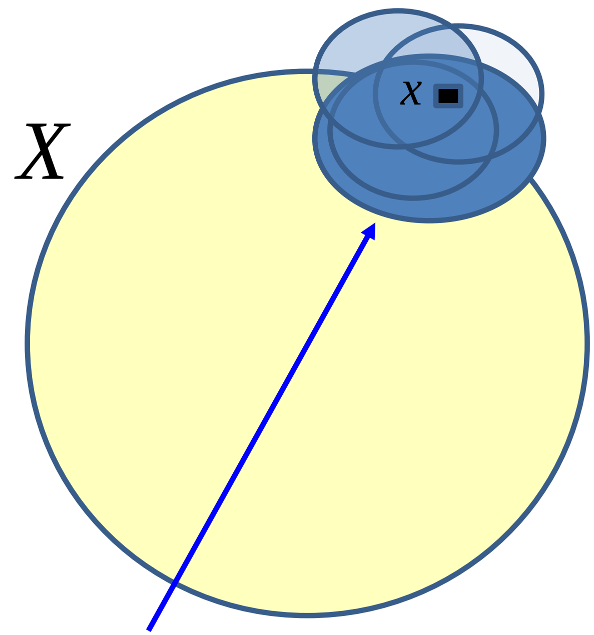
$$xIND(A)y\ \ iff\ \ \forall a \in A(a(x)\tau_a a(y))$$

$$xIND(A)y\ \ iff\ \ \forall a \in A(a(x) = a(y) \vee a(x) = * \vee a(y) = *)$$

GENERALIZATIONS OF ROUGH SETS: MANY POSSIBILITIES TO DEFINE APPROXIMATIONS

$$x \in \underline{\Im} X \quad iff \quad there\ is\ N_x \in \Im\ such\ that\ N_x \subseteq X$$





$$x \in \overline{\Im} X \quad iff \quad for\ any\ N_x \in \Im \quad N_x \cap X \neq \emptyset$$

GENERALIZATIONS OF ROUGH SETS

- Similarity (tolerance) Based Rough Set Approach;
- Variable Precision Rough Set Model
- Binary Relation Based Rough Sets;
- Neighborhood and Covering Rough Set Approach;
- Dominance Based Rough Set Approach;
- Probabilistic Rough Set Approach and its probabilistic extension called Variable Consistency Dominance Based Rough Set Approaches;
- Parameterized Rough Sets Based on Bayesian Confirmation Measures;
- Stochastic Rough Set Approach;
- Generalizations of Rough Set Approximation Operations;
- Hybridization of Rough Sets and Fuzzy Sets;
- Rough Sets on Abstract Algebraic Structures (e.g., lattices);
- …

J. Kacprzyk, W. Pedrycz (eds.), Handbook of Computational Intelligence, Springer, 2015 (part on rough sets).

UNCERTAINTY IN SELECTION (DISCOVERY) OF RELEVANT APPROXIMATION SPACE

A. Skowron, J. Stepaniuk, Generalized Approximation Spaces 1994

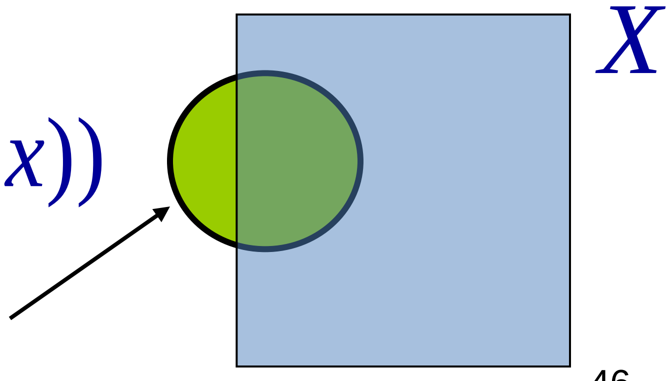
$$AS = (U, N, \nu)$$

$$N : U \rightarrow P(U)$$ neighborhood function

$$\nu : P(U) \times P(U) \rightarrow [0,1]$$ rough inclusion

partial function

$$x \rightarrow Inf(x) \rightarrow N(x) = Inf^{-1}(Inf(x))$$



neighborhood of x

APPROXIMATION SPACE

$$AS = (U, N, \nu)$$

$$LOW(AS, X) = \{x \in U : \nu(N(x), X) = 1\}$$

$$UPP(AS, X) = \{x \in U : \nu(N(x), X) > 0\}$$

<u>uncertainty in membership</u>: degree of membership of *x* into *X*

ROUGH MEREOLOGY

MEREOLOGY

St. LEŚNIEWSKI (1916)

x is_a_ part_of y

ROUGH MEREOLOGY

L. Polkowski and A. Skowron (1994-…)

x is_a_ part_of y in a degree

L. Polkowski, A. Skowron, Rough mereology, ISMIS'94, LNAI 869, Springer, 1994, 85-94

L. Polkowski, Reasonng by parts: An outline of rough mereology, Springer 2011

ROUGH SETS

- **ROUGH SETS DEFINED BY UPPER AND LOWER APPROXIMATIONS**

$$e.g.,\ rough\ sets\ are\ pairs\ of\ definable\ sets$$
$$(X,Y)\ \ where\ \ X \subseteq Y$$

- **AXIOMATIC APPROACH**

$$axioms\ for\ operations$$
$$\underline{B} : P(U) \rightarrow P(U)$$
$$\overline{B} : P(U) \rightarrow P(U)$$

RS
AND
DEDUCTIVE REASONING

RS and 3 valued logics

RS and (multi) modal logics

RS and multivalued logics

- partial order on truth values defined by different parts of boundary regions

RS and paraconsistent logics

…

UNCERTAINTY IN INFORMATION ABOUT APPROXIMATED CONCEPTS

ROUGH SETS AND INDUCTION

- **INDUCTIVE EXTENSIONS OF APPROXIMATION SPACES**
- **ADAPTIVE ROUGH SETS**

UNCERTAINTY IN CONCEPT DESCRIPTION

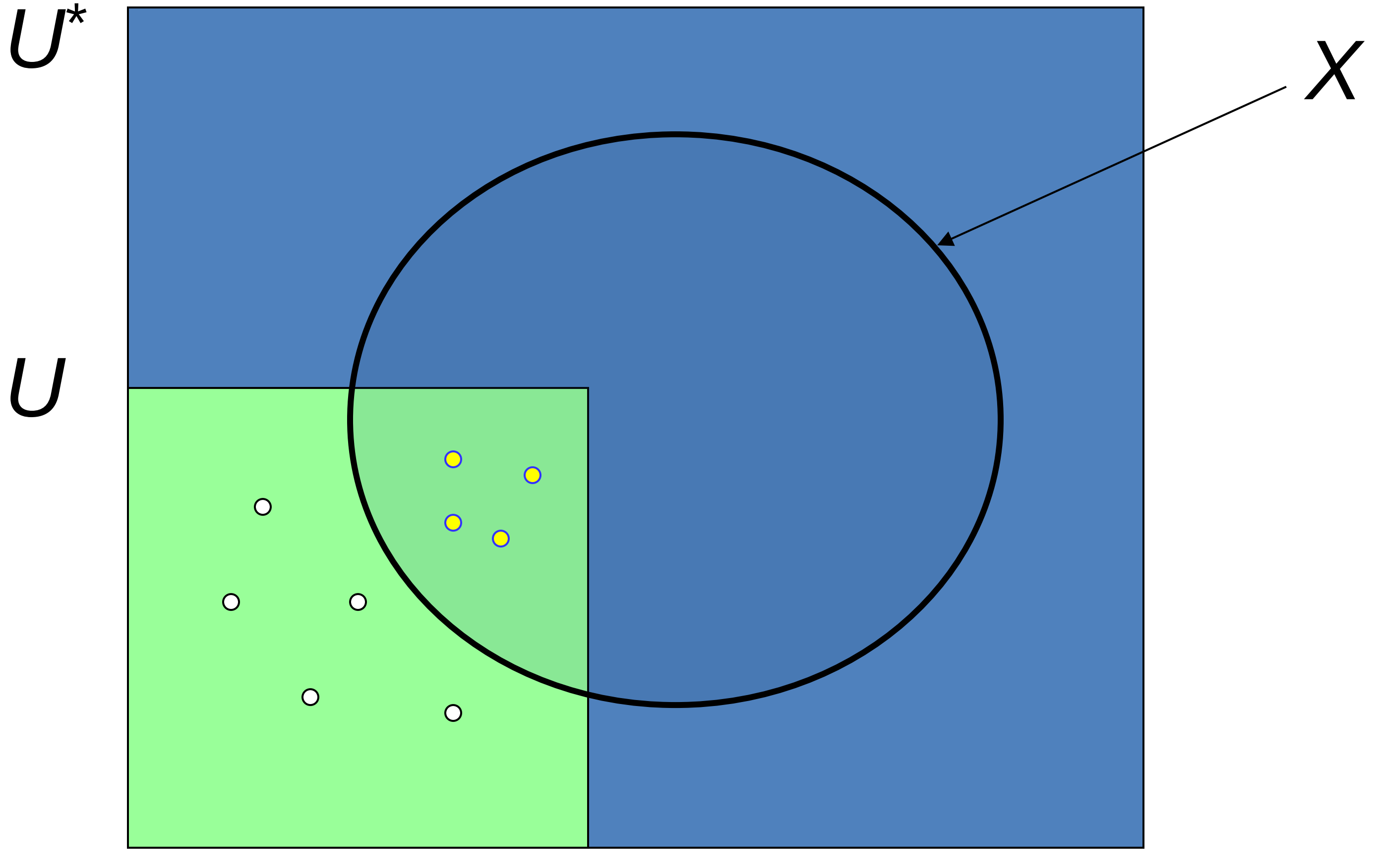
WHICH APPROXIMATION WE SHOULD SELECT?

DESCRIPTION VS REDUCTS

COST (DESCRIPTION LENGTH) VS MODEL QUALIY

MINIMUM DESCRIPTON LENGTH PRINCIPLE (MDL)

ROUGH SETS AND INDUCTON EXTENSIONS OF APPROXIMATION SPACES



FROM
RS IN DEDUCTIVE REASONING
TO
RS IN INDUCTIVE REASONING

RS AND INDUCTION

RS BASED CLASSIFIERS

ROUGH CLUSTERING, ROUGH-FUZZY CLUSTERING, …

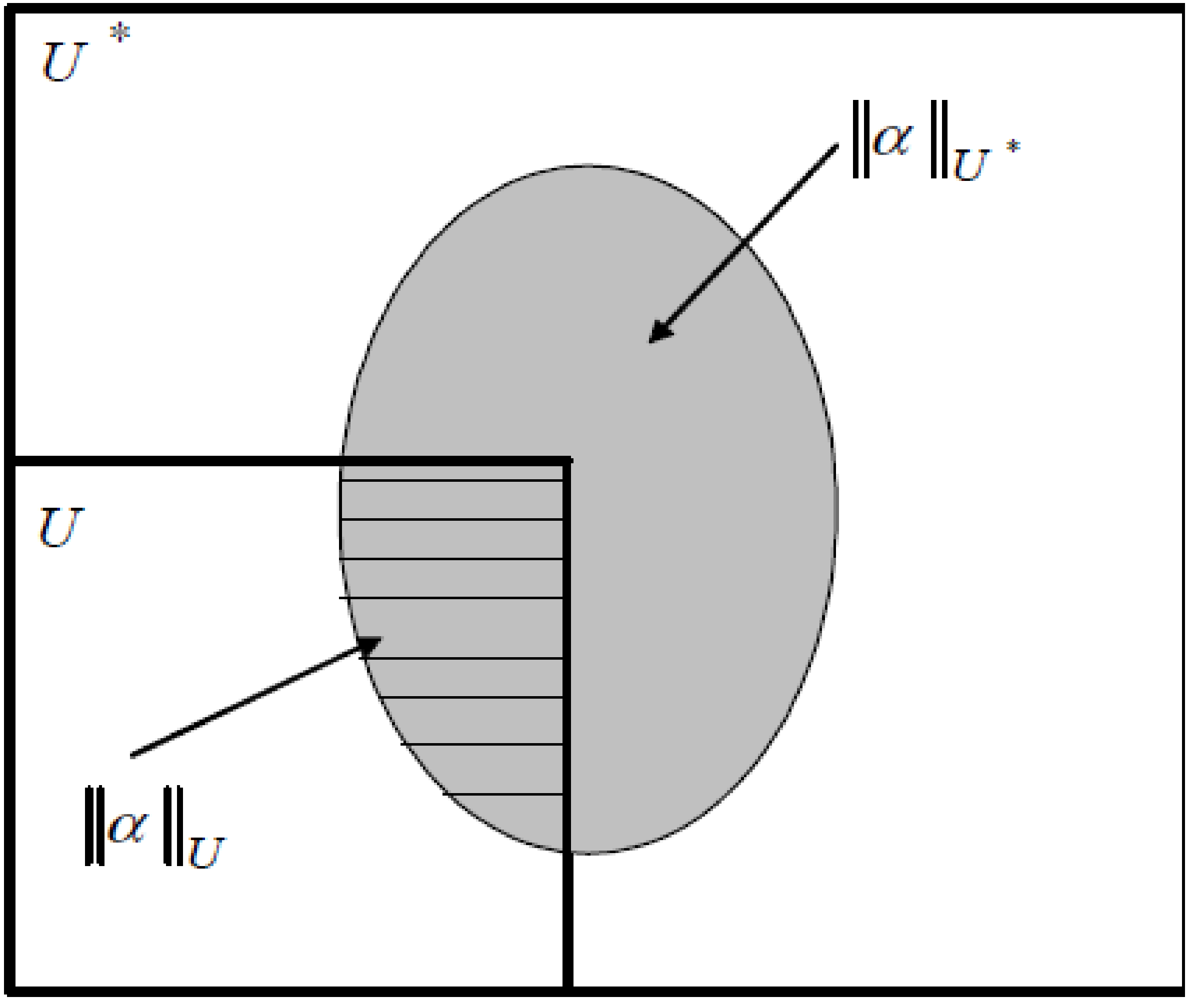
RS APPROACH TO DISCOVERY OF PROCESS MODELS FROM DATA

RS AND CONTEXT INDUCING

RS AND DISCOVERY OF HIERARCHY OF SATISFIABILIY RELATIONS IN HIERARCHICAL LEARNING (RS ONTOLOGY APPROXIMATION)

…

TWO SEMANTICS



APPROXIMATION EXTENSIONS: CLASSIFIERS

$$C \subseteq U^*, U \subseteq U^*$$

$$\chi_C(x) = \begin{cases} 1 & iff\ x \in U^* \\ 0 & otherwise \end{cases}$$

$$DT = (U, A, \chi_{C \cap U})$$

partial information about C

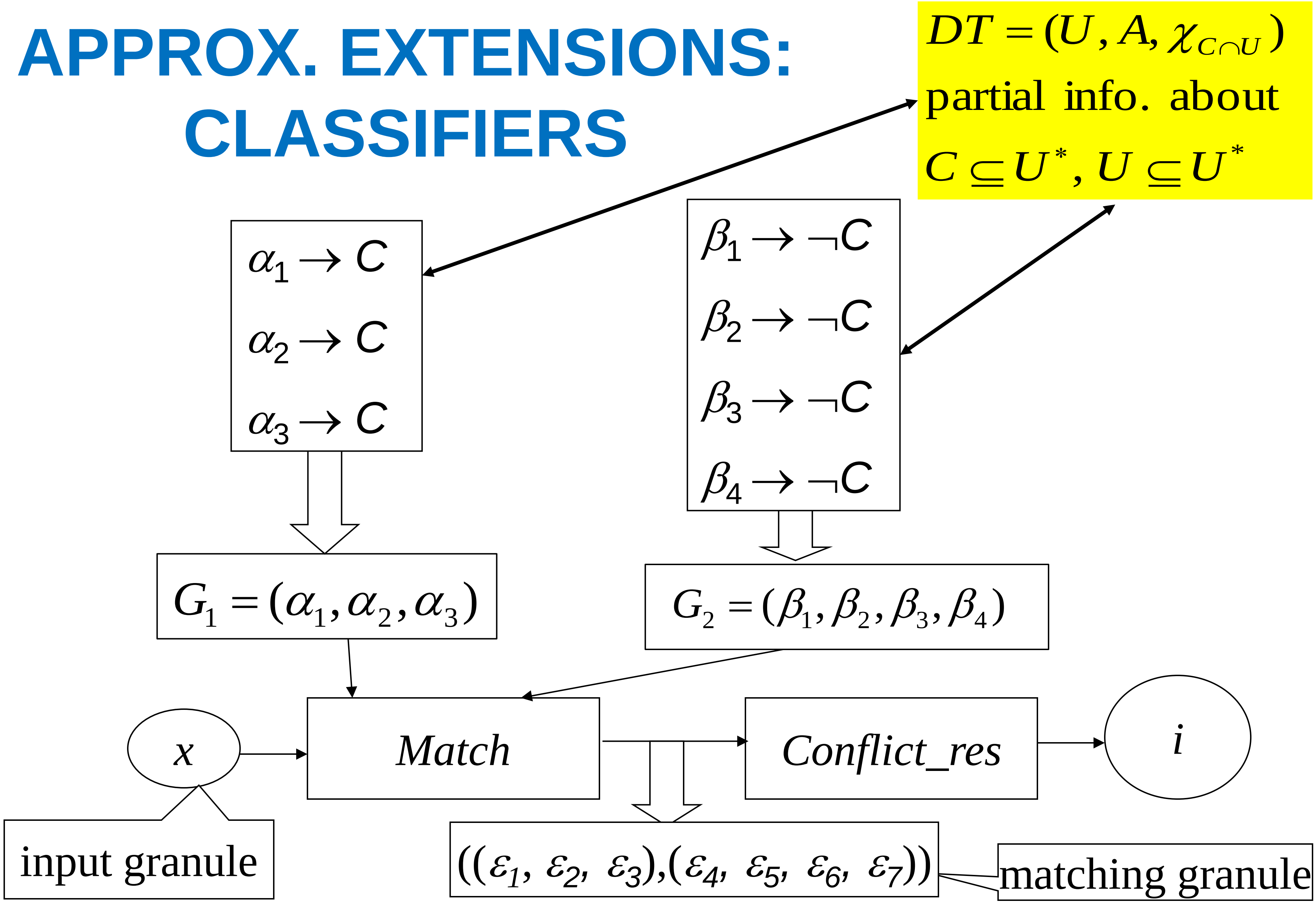
How to extend

$$\chi_{C \cap U} \text{ to } d^* : U^* \to L,\ where\ \{0,1\} \subseteq L$$

$$s.t.\ \ d^* \approx \chi_C ?$$

*d** approximates χ_C

APPROX. EXTENSIONS: CLASSIFIERS



Conflict_res (Match(Inf $_A$ ***(x),G***$_1$***,G***$_2$***))***

APPROXIMATION EXTENSIONS: CLASSIFIERS

Conflict_res **(*Match*(*x*,G_1,G_2))**

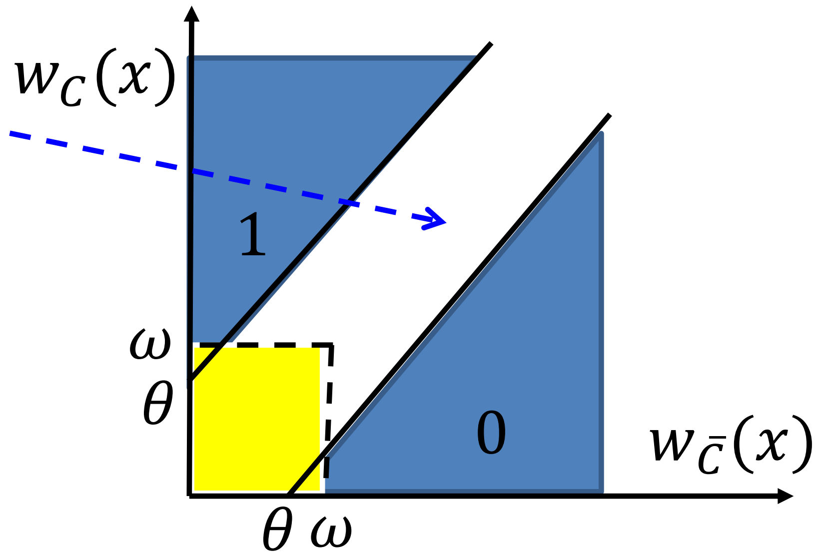
⇩

decision based on difference of weights:

$$w_C(x): \text{ weight of arguments for } C \text{ in case of } x$$

$$w_{\overline{C}}(x): \text{ weight of arguments against } C \text{ in case of } x$$

$$\mu_C(x) = \begin{cases} undefined & if \quad \max(w_C(x), w_{\bar{C}}(x)) \leq \omega \\ 0 & if \quad w_{\bar{C}}(x) - w_C(x) \geq \theta \; and \; w_{\bar{C}}(x) > \omega \\ 1 & if \quad w_C(x) - w_{\bar{C}}(x) \geq \theta \; and \; w_C(x) > \omega \\ \frac{\theta + w_C(x) - w_{\bar{C}}(x)}{2\theta} & otherwise \end{cases}$$



$$w_k(x) = \sum_{r \epsilon R_k(x)} strength(r)$$

$R_k(x)$ – the set of rules from $Match(A_d, x)$ for *k*

$$k \epsilon \{C, \bar{C}\}$$

APPROXIMATION EXTENSIONS: CLASSIFIERS

$$\underline{A}C = \{x \in U^* : \mu_C(x) = 1\}$$

$$\overline{A}C = \{x \in U^* : 0 < \mu_C(x) \leq 1 \vee$$

$$\mu_C(x) = undefined\}$$

$$BN_A(C) = \overline{A}C \setminus \underline{A}C =$$

$$= \{x \in U^* : 0 < \mu_C(x) < 1 \vee$$

$$\mu_C(x) = undefined\}$$

ROUGH SETS
AND
VAGUE CONCEPTS

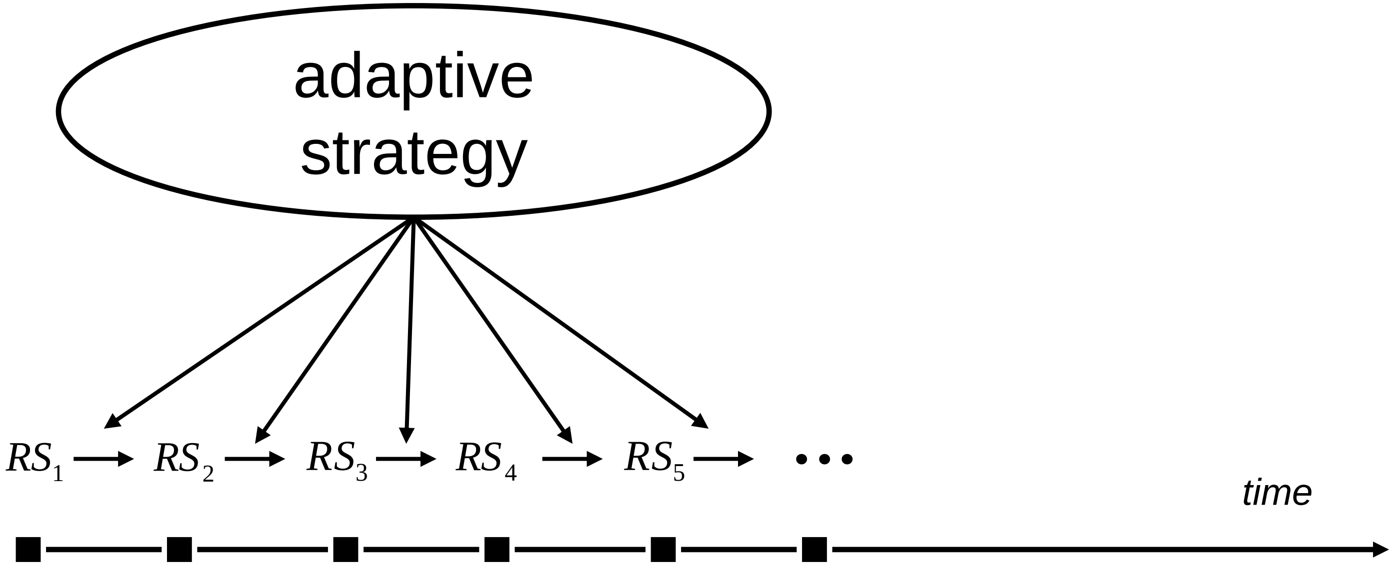
VAGUENESS IN PHILOSOPHY

Discussion on vague (imprecise) concepts includes the following :

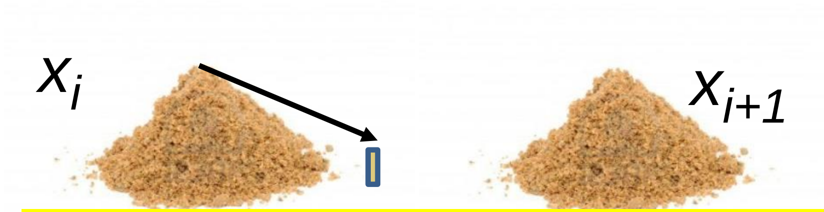
1. The presence of borderline cases.
2. Boundary regions of vague concepts are not crisp.
3. Vague concepts are susceptible to sorites paradoxes.

Keefe, R. (2000) Theories of Vagueness. Cambridge Studies in Philosophy, Cambridge, UK)

ROUGH SETS AND VAGUE CONCEPTS
ADAPTIVE ROUGH SETS



Boundary regions of vague concepts are not crisp
ADAPTIVE ROUGH SETS



SORITES PARADOXES

$$x_1,\dots,x_i,x_{i+1},\dots,x_N;\ \mu_C(x_1)=1,\ \mu_C(x_N)=0$$

$$\begin{aligned}
&If\\
&x_i \in \underline{A}C \to x_{i+1} \in \overline{A}C\\
&w_C(x_{i+1}) \le w_C(x_i),\ w_{\overline{C}}(x_{i+1}) \ge w_{\overline{C}}(x_i)\\
&then\\
&there\ exists\ i_0:\ x_{i_0} \in BN_A(C)\\
&x_i \in BN_A(C) \to x_{i+1} \in \overline{A}(U^* \setminus C)\\
&x_i \in \underline{A}(U^* \setminus C) \to x_{i+1} \in \underline{A}(U^* \setminus C)
\end{aligned}$$

SORITES PARADOXES

One can add a condition

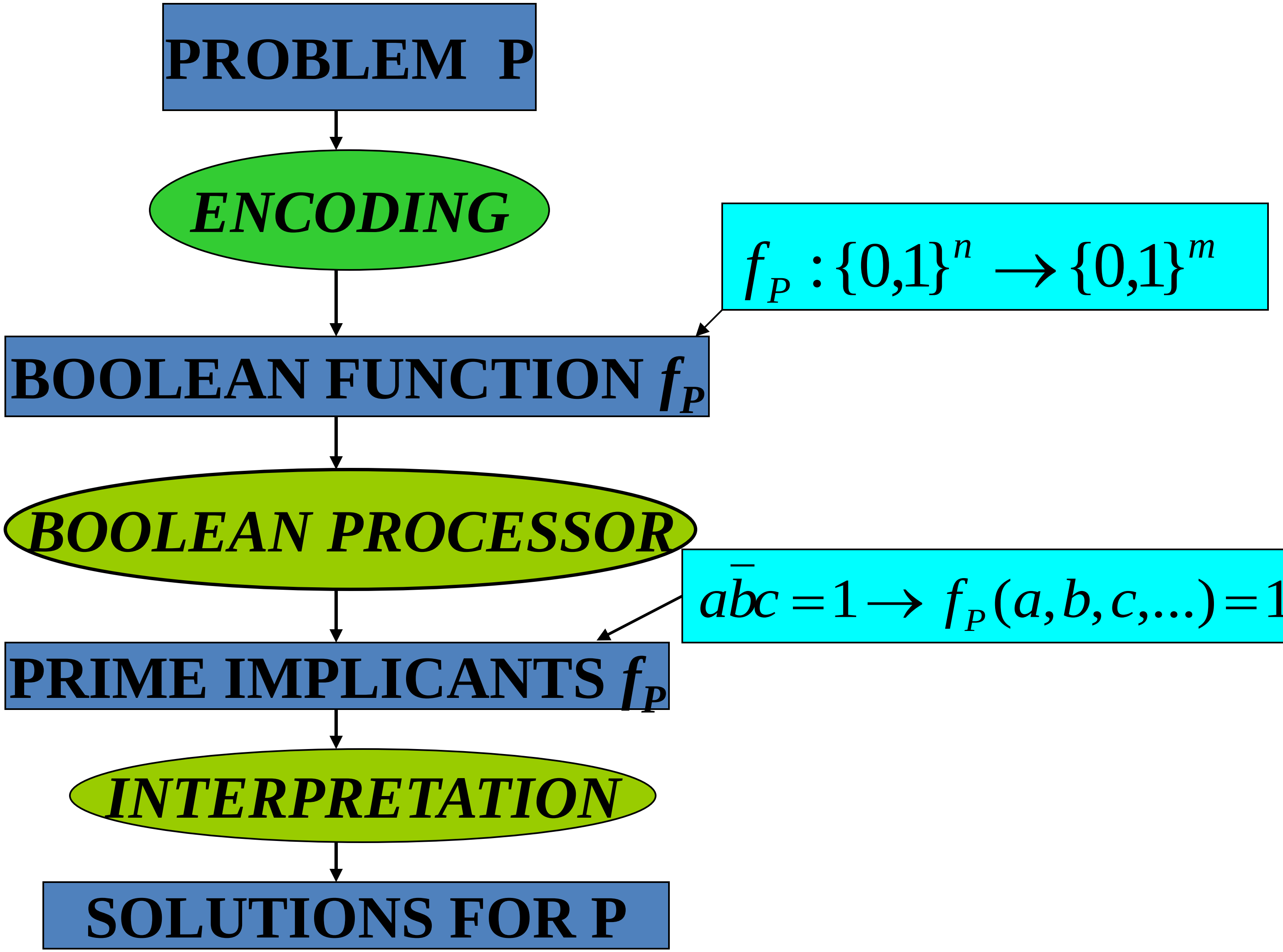
$$w_C(x_i) - w_C(x_{i+1}) \leq \partial,$$
$$w_{\overline{C}}(x_{i+1}) - w_{\overline{C}}(x_i) \leq \partial.$$

where ∂ is a given threshold bounding jumps in degrees of memberships of x_i and x_{i+1}.

RELATIONSHIPS OF ROUGH SETS WITH BOOLEAN REASONING

BOOLEAN REASONING

George Boole (1815-1864)



BOOLEAN REASONING

- **Rough Sets and Boolean Reasoning**
  - **Reducts in information systems**
  - **Decision reducts**
  - **Local reducts relative to objects**
  - **Discretization**
  - **Symbolic value grouping**
  - **Approximate reducts and association rules**

BOOLEAN REASONING

DISCERNIBILITY CONSTRAINTS

TO BE PRESERVED

CAN BE ENCODED BY MEANS OF

BOOLEAN FUNCTIONS

RELEVANT

FOR BOOLEAN REASONING

BOOLEAN REASONING FOR COMPUTING REDUCTS IN INFORMATION SYSTEMS

REDUCTS IN *IS*

$$IS = (U, A)$$

$$Discernibility\ \ matrix$$

$$M(IS) = (c_{ij})_{n\times n} : c_{ij} = \{a \in A : a(x_i) \neq a(x_j)\}$$

$$Discernibility\ \ function$$

$$f_{IS}(a_1,\ldots,a_m) = \wedge\{\vee c_{ij} : 1 \leq i < j \leq n, c_{ij} \neq \emptyset\}$$

$$a_{i_1} \wedge \ldots \wedge a_{i_k}\ is\ a\ prime\ implicant\ of\ \ f_{IS}$$

$$iff\ \ \{a_{i_1},\ldots,a_{i_k}\} \in RED(IS)$$

REDUCTS IN *IS*

		x_i	
	…		…
x_j		a,b,c	
	…	c,e	…

$$(a \vee b \vee c) \wedge (c \vee e) \wedge \ldots$$

$$(ae \vee be \vee c) \wedge (\ldots) \wedge \ldots$$

BOOLEAN REASONING FOR COMPUTING DECISION REDUCTS

DT	a	b	...	d	∂_A
...					
x_i	1	2	1	1	
...					
x_j	2	0	0	0	
...					

		x_i	
	...	...	...
x_j	...	c_{ij}^{DT}	...

$$c_{ij}^{DT} = \begin{cases} c_{ij} & \text{if } \partial_A(x_i) \neq \partial_A(x_j) \\ \varnothing & \text{otherwise} \end{cases}$$

$$DT = (U, A, d)$$

$$\downarrow$$

Discernibility matrix

$$M(DT) = (c_{ij}^{DT})_{n\times n}$$

$$c_{ij}^{DT} = \begin{cases} \{a \in A : a(x_i) \neq a(x_j)\} & if \ \partial_A(x_i) \neq \partial_A(x_j) \\ \varnothing & otherwise \end{cases}$$

$$\downarrow$$

Discernibility function

$$f_{DT}(a_1, \ldots, a_m) = \wedge\{\vee c_{ij}^{DT} : 1 \leq i < j \leq n, c_{ij}^{DT} \neq \varnothing\}$$

$$\downarrow$$

$a_{i_1} \wedge \ldots \wedge a_{i_k}$ *is a prime implicant of* f_{DT}

iff $\{a_{i_1}, \ldots, a_{i_k}\} \in RED(DT)$

AN EXAMPLE: DECISION REDUCTS & CORE

Decision table

U	*Headache*	*Muscle pain*	*Temp.*	*Flu*
u1	Yes	Yes	Normal	No
u2	Yes	Yes	High	Yes
u3	Yes	Yes	Very-high	Yes
u4	No	Yes	Normal	No
u5	No	No	High	No
u6	No	Yes	Very-high	Yes

Discernibility matrix

	u2	*u3*	*u6*
u1	*Temp.*	*Temp.*	*Headache, Temp.*
u4	*Headache, Temp.*	*Headache, Temp.*	*Temp.*
u5	*Headache, Muscle pain*	*Headache, Muscle pain, Temp.*	*Muscle pain, Temp.*

Discernibility fuction

$$(Temp.) \wedge (Temp.) \wedge (Headache \vee Temp.) \wedge$$

$$(Headache \vee Temp.) \wedge (Headache \vee Temp.) \wedge (Temp.) \wedge$$

$$(Headache \vee Muscle\ pain) \wedge (Headache \vee Muscle\ pain \vee Temp.) \wedge (Muscle\ pain \vee Temp.)$$

$$\Leftrightarrow$$

$$Temp. \wedge (Headache \vee Muscle\ pain)$$

$$\Leftrightarrow$$

$$(Temp. \wedge Headache) \vee (Temp. \wedge Muscle\ pain)$$

AN EXAMPLE: DECISION REDUCTS & CORE

U	*Headache*	*Muscle pain*	*Temp.*	*Flu*
u1	Yes	Yes	Normal	No
u2	Yes	Yes	High	Yes
u3	Yes	Yes	Very-high	Yes
u4	No	Yes	Normal	No
u5	No	No	High	No
u6	No	Yes	Very-high	Yes

***Reduct1* = {*Muscle-pain,Temp.*}**

U	*Muscle pain*	*Temp.*	*Flu*
u1,u4	Yes	Normal	No
u2	Yes	High	Yes
u3,u6	Yes	Very-high	Yes
u5	No	High	No

***Reduct2* = {*Headache, Temp.*}**

U	*Headache*	*Temp.*	*Flu*
u1	Yes	Normal	No
u2	Yes	High	Yes
u3	Yes	Very-high	Yes
u4	No	Normal	No
u5	No	High	No
u6	No	Very-high	Yes

***CORE* = {*Headache,Temp.*} $\cap$ {*MusclePain, Temp.*} = {Temp.}**

BOOLEAN REASONING FOR COMPUTING LOCAL REDUCTS RELATIVE TO OBJECTS

LOCAL REDUCTS

	x_i
	…
x_j	$c_j = \begin{cases} c_{ij} & \text{if } \partial_{DT}(x_i) \neq \partial_{DT}(x_j) \\ \varnothing & \text{otherwise} \end{cases}$
	…

only column for a given object x_i is considered

DECISION RULES FROM LOCAL REDUCTS

Any local reduct B relative to a given object defines a minimal decision rule.

$\{a,b\}$ local reduct for x

	a	b	$\cdots$	d
x	1	2		0

drooping any descriptor makes the rule inconsistent with the generalized decision

$a=1 \wedge b=2 \Rightarrow d=0$ minimal decision rule for x

Remark: Illustration for consistent decision system

DISCRETIZATION

DISCRETIZATION

- In the discretization of a decision table $DT = (U, A, d)$, where $V_a = [v_a, w_a)$ is an interval of real-valued values, we search for a partition P_a of V_a for any $a \in A$.
- Any partition of V_a is defined by a sequence of *cuts* $v_1 < v_2 < \ldots < v_k$ from V_a.
- Any family of partitions $\{P_a\}_{a \in A}$ can be identified with a set of cuts.

DISCRETIZATION

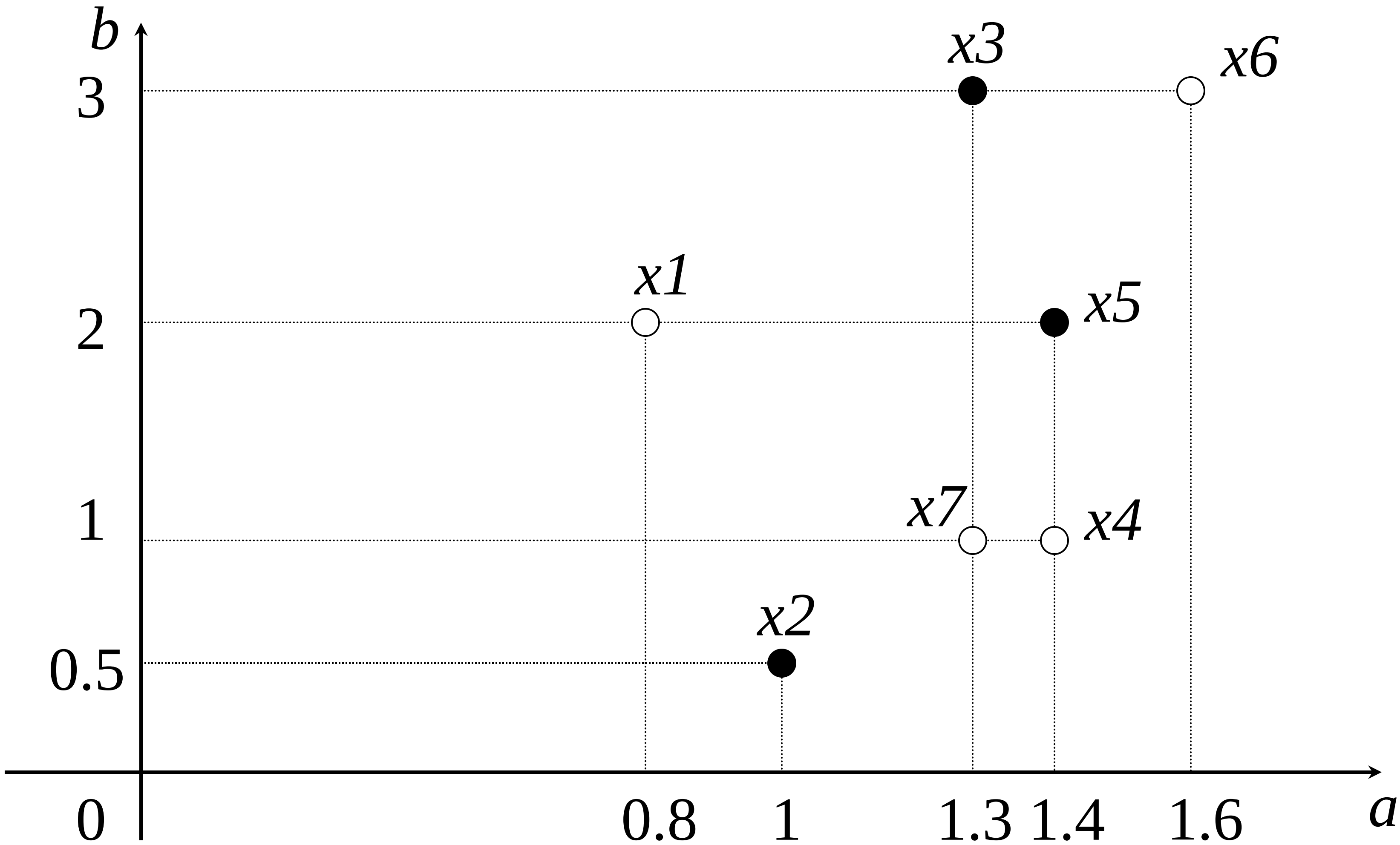
In the discretization process, we search for a set of cuts satisfying some natural conditions.

DT	*a*	*b*	*d*
x1	0.8	2	1
x2	1	0.5	0
x3	1.3	3	0
x4	1.4	1	1
x5	1.4	2	0
x6	1.6	3	1
x7	1.3	1	1

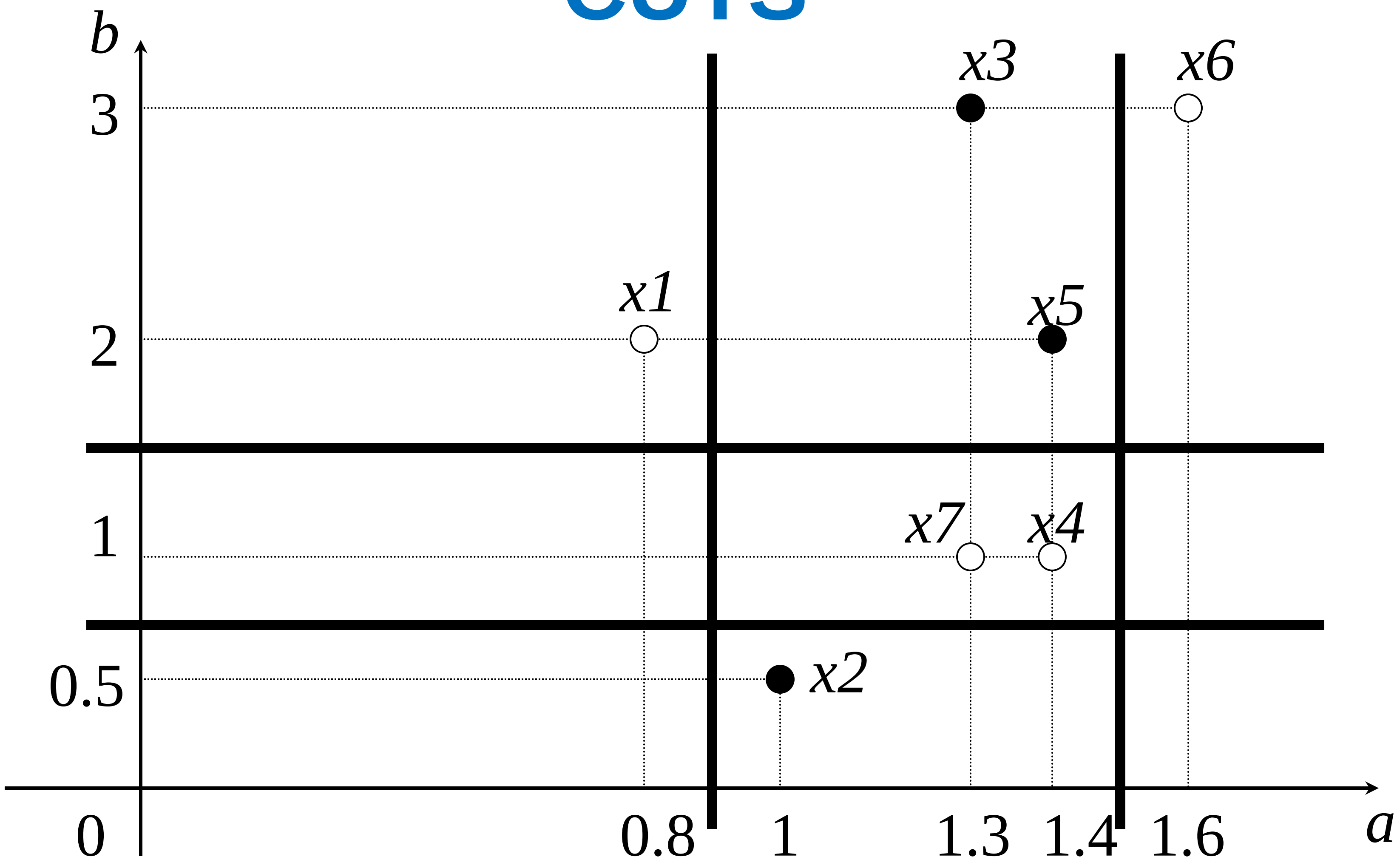
$$P = \{(a, 0.9), (a, 1.5), (b, 0.75), (b, 1.5)\}$$

DT	a^P	b^P	*d*
x1	0	2	1
x2	1	0	0
x3	1	2	0
x4	1	1	1
x5	1	2	0
x6	2	2	1
x7	1	1	1

A GEOMETRICAL REPRESENTATION OF DATA



A GEOMETRICAL REPRESENTATION OF DATA AND CUTS



DISCRETIZATION

- The sets of possible values of *a* and *b* are defined by $V_a = [0,2); \; V_b = [0,4).$

- The sets of values of *a* and *b* on objects from *U* are given by

  $a(U) = \{0.8, 1, 1.3, 1.4, 1.6\};$

  $b(U) = \{0.5, 1, 2, 3\}.$

The discretization process returns a partition of the value sets of conditional attributes into intervals.

DISCRETIZATION PROCESS

- ***Step 1***: define a set of Boolean variables,

$$BV(U) = \{p_1^a, p_2^a, p_3^a, p_4^a, p_1^b, p_2^b, p_3^b\}$$

where

p_1^a corresponds to the interval [0.8, 1) of *a*

p_2^a corresponds to the interval [1, 1.3) of *a*

p_3^a corresponds to the interval [1.3, 1.4) of *a*

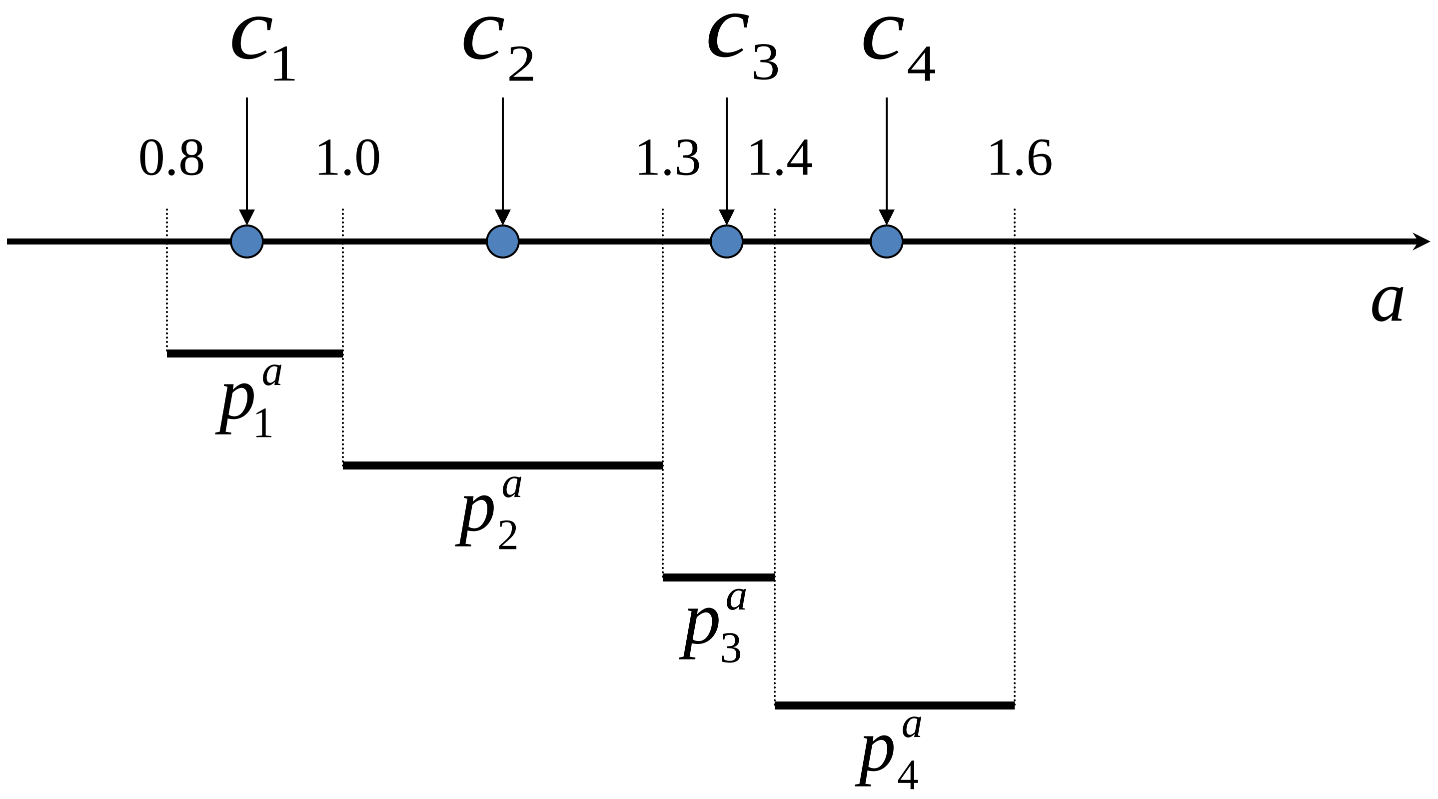
p_4^a corresponds to the interval [1.4, 1.6) of *a*

p_1^b corresponds to the interval [0.5, 1) of *b*

p_2^b corresponds to the interval [1, 2) of *b*

p_3^b corresponds to the interval [2, 3) of *b*

THE SET OF CUTS ON ATTRIBUTE *a*



DISCRETIZATION PROCESS

Step 2: Let $DT = (U, A, d)$ be a decision table, p_k^a be a propositional variable corresponding to the interval $[v_k^a, v_{k+1}^a)$ for any *k* and $a \in A$.

Create a new decision table by using the set of Boolean variables defined in *Step 1*.

A NEW TABLE DEFINED IN *Step 2*

U^*	p_1^a	p_2^a	p_3^a	p_4^a	p_1^b	p_2^b	p_3^b
$(x1,x2)$	1	0	0	0	1	1	0
$(x1,x3)$	1	1	0	0	0	0	1
$(x1,x5)$	1	1	1	0	0	0	0
$(x4,x2)$	0	1	1	0	1	0	0
$(x4,x3)$	0	0	1	0	0	1	1
$(x4,x5)$	0	0	0	0	0	1	0
$(x6,x2)$	0	1	1	1	1	1	1
$(x6,x3)$	0	0	1	1	0	0	0
$(x6,x5)$	0	0	0	1	0	0	1
$(x7,x2)$	0	1	0	0	1	0	0
$(x7,x3)$	0	0	0	0	0	1	0
$(x7,x5)$	0	0	1	0	0	1	0

THE DISCERNIBILITY FORMULA

- The discernibility formula

$$\psi(x_1, x_2) = p_1^a \vee p_1^b \vee p_2^b$$

means that in order to discern object *x1* and *x2*, at least one of the following cuts must be set,

a cut between *a*(0.8) and *a*(1)
a cut between *b*(0.5) and *b*(1)
a cut between *b*(1) and *b*(2).

THE DISCERNIBILITY FORMULAE FOR ALL DIFFERENT PAIRS

$$\psi(x_1, x_2) = p_1^a \vee p_1^b \vee p_2^b$$
$$\psi(x_1, x_3) = p_1^a \vee p_2^a \vee p_3^b$$
$$\psi(x_1, x_5) = p_1^a \vee p_2^a \vee p_3^a$$
$$\psi(x_4, x_2) = p_2^a \vee p_3^a \vee p_1^b$$
$$\psi(x_4, x_3) = p_2^a \vee p_2^b \vee p_3^b$$
$$\psi(x_4, x_5) = p_2^b$$

THE DISCERNIBILITY FORMULAE FOR ALL DIFFERENT PAIRS

$$\psi(x_6, x_2) = p_2^a \vee p_3^a \vee p_4^a \vee p_1^b \vee p_2^b \vee p_3^b$$

$$\psi(x_6, x_3) = p_3^a \vee p_4^a$$

$$\psi(x_6, x_5) = p_4^a \vee p_3^b$$

$$\psi(x_7, x_2) = p_2^a \vee p_1^b$$

$$\psi(x_7, x_3) = p_2^b \vee p_3^b$$

$$\psi(x_7, x_5) = p_3^a \vee p_2^b$$

DISCRETIZATION PROCESS

- ***Step 3:*** Find the minimal subset of the set *P* of propositional variables that discerns all objects from different decision classes.

  The discernibility boolean propositional formula is defined as follows

$$\Phi^U = \wedge\{\psi(x_i, x_j) : d(x_i) \neq d(x_j)\}.$$

THE DISCERNIBILITY FORMULA IN CNF FORM

$$\begin{aligned}\Phi^U = {} & (p_1^a \vee p_1^b \vee p_2^b) \wedge (p_1^a \vee p_2^a \vee p_3^b) \\ & \wedge (p_2^a \vee p_3^a \vee p_1^b) \wedge (p_2^a \vee p_2^b \vee p_3^b) \\ & \wedge (p_2^a \vee p_3^a \vee p_4^a \vee p_1^b \vee p_2^b \vee p_3^b) \\ & \wedge (p_3^a \vee p_4^a) \wedge (p_4^a \vee p_3^b) \wedge (p_2^a \vee p_1^b) \\ & \wedge (p_2^b \vee p_3^b) \wedge (p_3^a \vee p_2^b) \wedge p_2^b.\end{aligned}$$

THE DISCERNIBILITY FORMULA IN DNF FORM

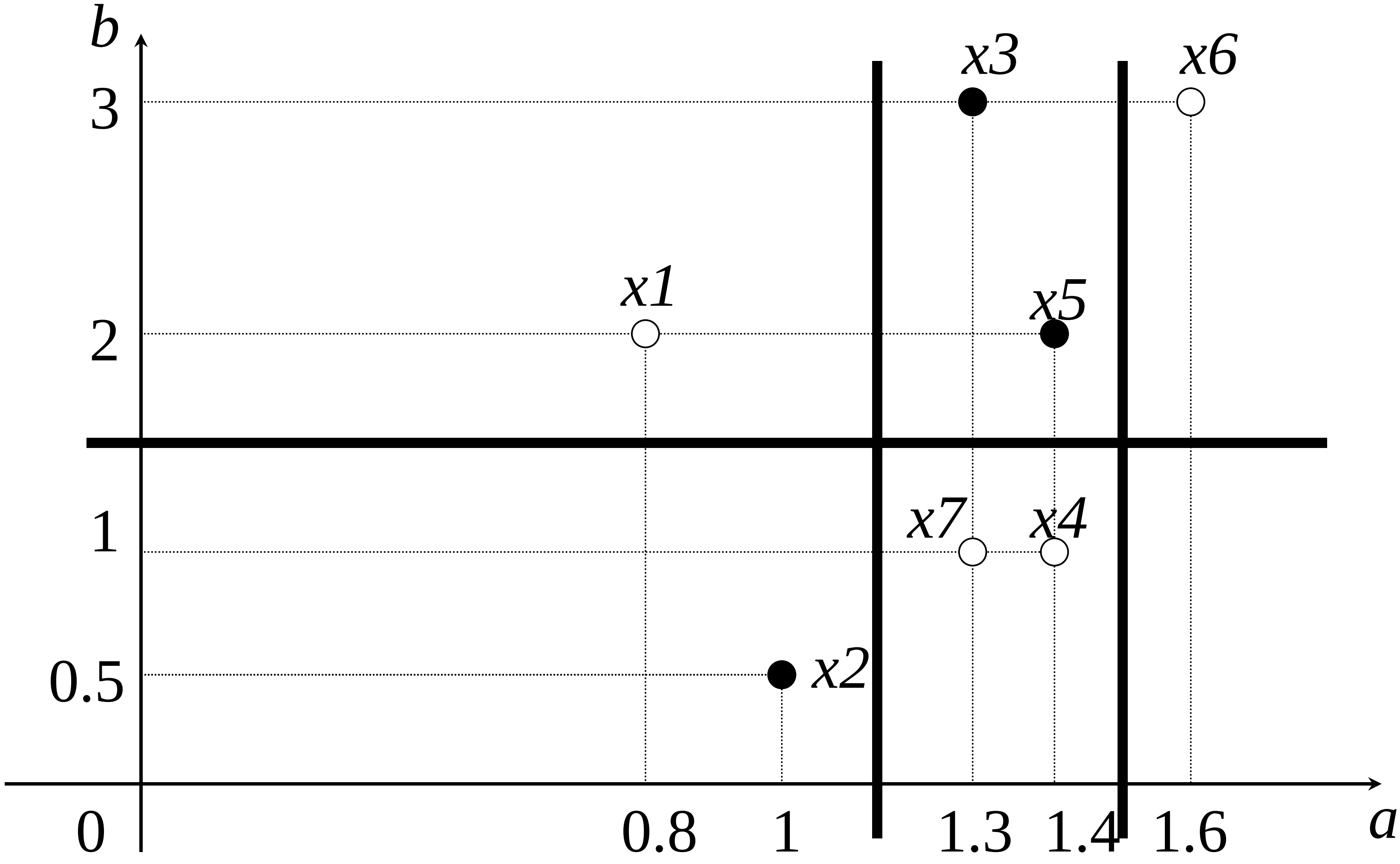
- We obtain four prime implicants,

$$\Phi^U = (p_2^a \wedge p_4^a \wedge p_2^b) \vee (p_2^a \wedge p_3^a \wedge p_2^b \wedge p_3^b)$$
$$\vee (p_3^a \wedge p_1^b \wedge p_2^b \wedge p_3^b) \vee (p_1^a \wedge p_4^a \wedge p_1^b \wedge p_2^b).$$

$\{p_2^a, p_4^a, p_2^b\}$ is the optimal result,

because it is the minimal subset of P.

THE MINIMAL SET OF CUTS FOR THE SAMPLE *DT*



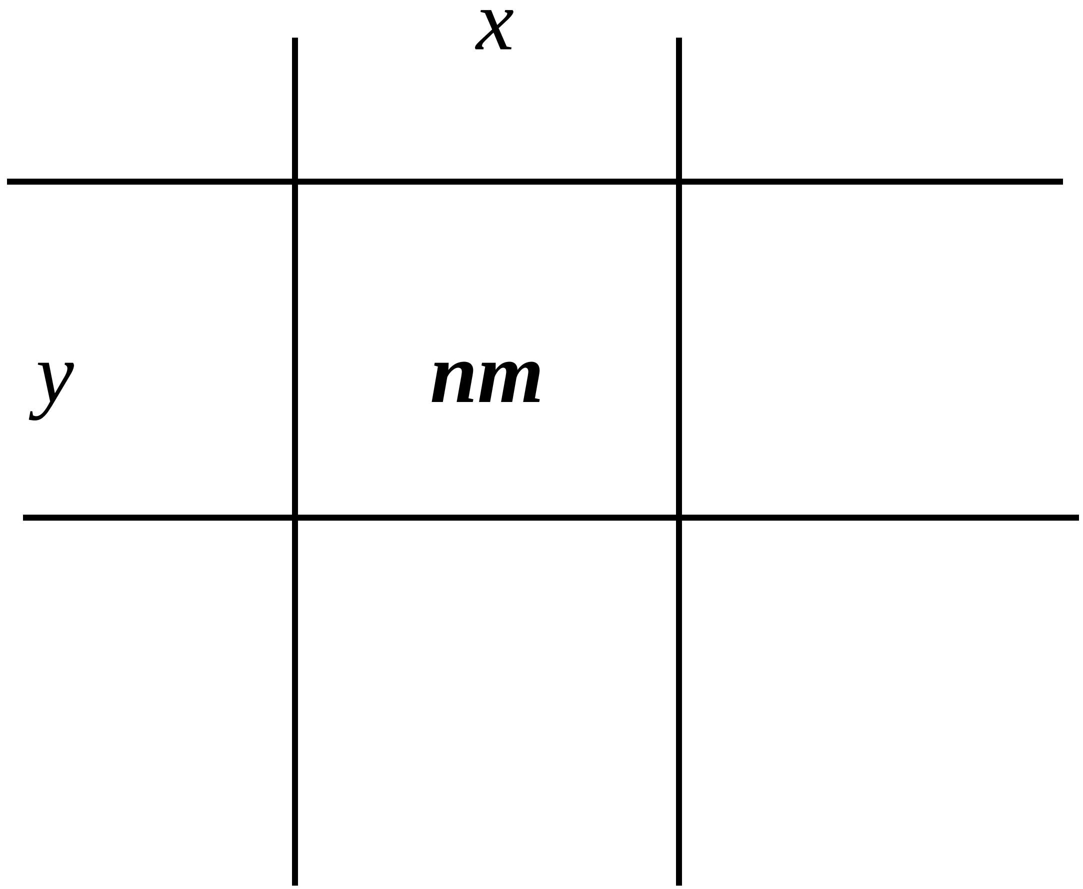
A RESULT

DT	*a*	*b*	*d*
x1	0.8	2	1
x2	1	0.5	0
x3	1.3	3	0
x4	1.4	1	1
x5	1.4	2	0
x6	1.6	3	1
x7	1.3	1	1

$$P = \{(a, 1.2), (a, 1.5), (b, 1.5)\}$$

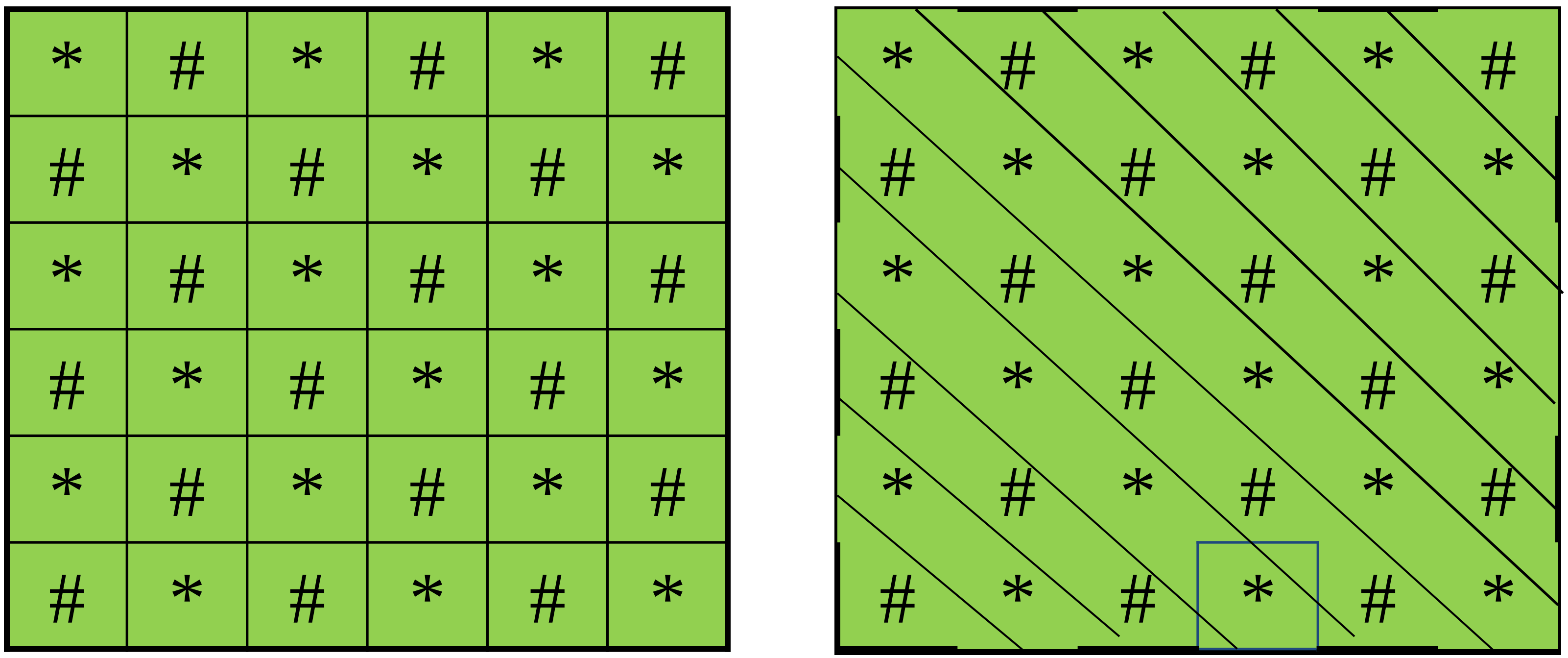
A	a^P	b^P	*d*
x1	0	1	1
x2	0	0	0
x3	1	1	0
x4	1	0	1
x5	1	1	0
x6	2	1	1
u7	1	0	1

DISCRETIZATION

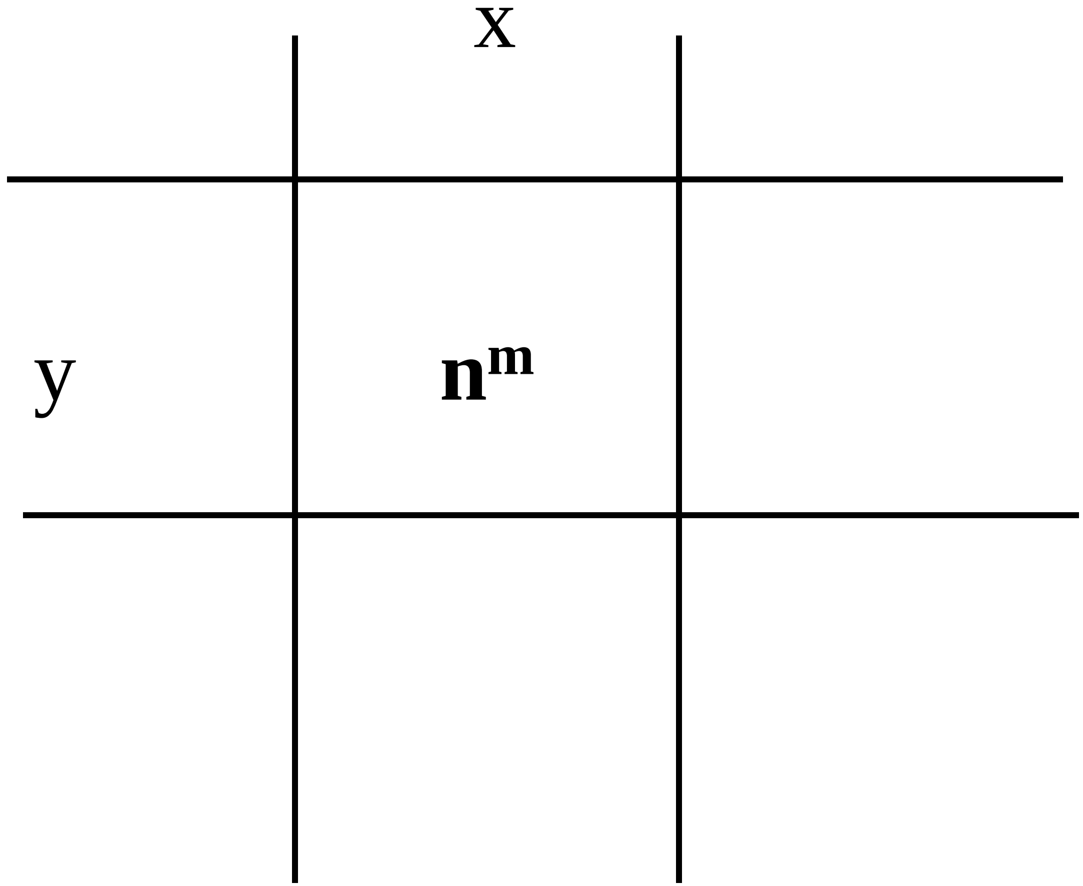


SEARCHING FOR FEARURES DEFINED BY HYPERPLANES

HYPERPLANES

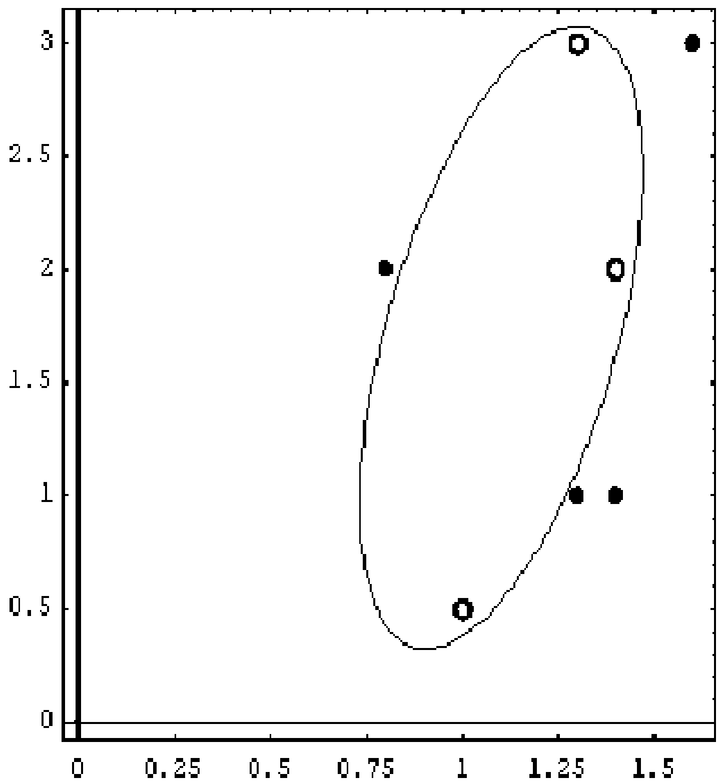


HYPERPLANES



SEARCHING FOR FEARURES DEFINED BY HIGHER ORDER SURFACES

SECOND ORDER SURFACES



$\mathbf{A}$	a	b	d
u_1	0.8	2	1
u_2	1	0.5	0
u_3	1.3	3	0
u_4	1.4	1	1
u_5	1.4	2	0
u_6	1.6	3	1
u_7	1.3	1	1

(a)

$\mathbf{A}^2$	a	b	a^2	ab	b^2	H^*	d
u_1	0.8	2	0.64	1.6	4	0.36	1
u_2	1	0.5	1	0.5	0.25	-0.25	0
u_3	1.3	3	1.69	3.9	9	-0.14	0
u_4	1.4	1	1.96	1.4	1	1.24	1
u_5	1.4	2	1.96	2.8	4	-0.36	0
u_6	1.6	3	2.56	4.8	9	1.24	1
u_7	1.3	1	1.69	1.3	1	0.26	1

$$H^* = -24a + b + 14a^2 - 4ab + b^2 + 11$$

SYMBOLIC VALUE GROUPING

SYMBOLIC VALUE GROUPING

DT	a	b	d
u_1	a_1	b_1	0
u_2	a_1	b_2	0
u_3	a_2	b_3	0
u_4	a_3	b_1	0
u_5	a_1	b_4	1
u_6	a_2	b_2	1
u_7	a_2	b_1	1
u_8	a_4	b_2	1
u_9	a_3	b_4	1
u_{10}	a_2	b_5	1

$\mathcal{M}(DT)$	u_1	u_2	u_3	u_4
u_5	$b_{b_4}^{b_1}$	$b_{b_4}^{b_2}$	$a_{a_2}^{a_1}, b_{b_4}^{b_3}$	$a_{a_3}^{a_1}, b_{b_4}^{b_1}$
u_6	$a_{a_2}^{a_1}, b_{b_2}^{b_1}$	$a_{a_2}^{a_1}$	$b_{b_3}^{b_2}$	$a_{a_3}^{a_2}, b_{b_2}^{b_1}$
u_7	$a_{a_2}^{a_1}$	$a_{a_2}^{a_1}, b_{b_2}^{b_1}$	$b_{b_3}^{b_1}$	$a_{a_3}^{a_2}$
u_8	$a_{a_4}^{a_1}, b_{b_2}^{b_1}$	$a_{a_4}^{a_1}$	$a_{a_4}^{a_2}, b_{b_3}^{b_2}$	$a_{a_4}^{a_3}, b_{b_2}^{b_1}$
u_9	$a_{a_3}^{a_1}, b_{b_4}^{b_1}$	$a_{a_3}^{a_1}, b_{b_4}^{b_2}$	$a_{a_3}^{a_2}, b_{b_4}^{b_3}$	$b_{b_4}^{b_1}$
u_{10}	$a_{a_2}^{a_1}, b_{b_5}^{b_1}$	$a_{a_2}^{a_1}, b_{b_5}^{b_2}$	$b_{b_5}^{b_3}$	$a_{a_3}^{a_2}, b_{b_5}^{b_1}$

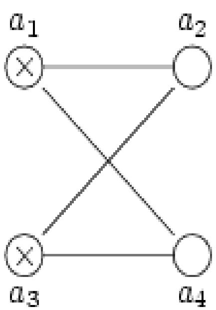
SYMBOLIC VALUE GROUPING

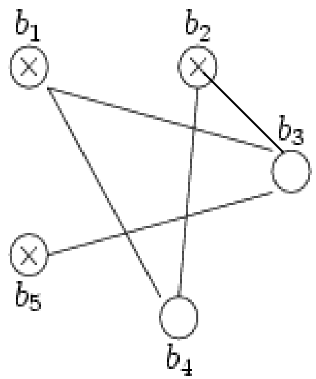
$$
\begin{aligned}
& b^{b_1}_{b_4} \wedge b^{b_2}_{b_4} \wedge (a^{a_1}_{a_2} \vee b^{b_3}_{b_4}) \wedge (a^{a_1}_{a_3} \vee b^{b_1}_{b_4}) \wedge \\
& (a^{a_1}_{a_2} \vee b^{b_1}_{b_2}) \wedge a^{a_1}_{a_2} \wedge b^{b_2}_{b_3} \wedge (a^{a_2}_{a_3} \vee b^{b_1}_{b_2}) \wedge \\
& a^{a_1}_{a_2} \wedge (a^{a_1}_{a_2} \vee b^{b_1}_{b_2}) \wedge b^{b_1}_{b_3} \wedge a^{a_2}_{a_3} \wedge \\
& (a^{a_1}_{a_4} \vee b^{b_1}_{b_2}) \wedge a^{a_1}_{a_4} \wedge (a^{a_2}_{a_4} \vee b^{b_2}_{b_3}) \wedge (a^{a_3}_{a_4} \vee b^{b_1}_{b_2}) \wedge \\
& (a^{a_1}_{a_3} \vee b^{b_1}_{b_4}) \wedge (a^{a_1}_{a_3} \vee b^{b_2}_{b_4}) \wedge (a^{a_2}_{a_3} \vee b^{b_3}_{b_4}) \wedge b^{b_1}_{b_4} \wedge \\
& (a^{a_1}_{a_2} \vee b^{b_1}_{b_5}) \wedge (a^{a_1}_{a_2} \vee b^{b_2}_{b_5}) \wedge b^{b_3}_{b_5} \wedge (a^{a_2}_{a_3} \vee b^{b_1}_{b_5}).
\end{aligned}
$$

$$I \equiv a^{a_1}_{a_2} \wedge a^{a_2}_{a_3} \wedge a^{a_1}_{a_4} \wedge a^{a_3}_{a_4} \wedge b^{b_1}_{b_4} \wedge b^{b_2}_{b_4} \wedge b^{b_2}_{b_3} \wedge b^{b_1}_{b_3} \wedge b^{b_3}_{b_5}$$

SYMBOLIC VALUE GROUPING

$$I \equiv a^{a_1}_{a_2} \wedge a^{a_2}_{a_3} \wedge a^{a_1}_{a_4} \wedge a^{a_3}_{a_4} \wedge b^{b_1}_{b_4} \wedge b^{b_2}_{b_4} \wedge b^{b_2}_{b_3} \wedge b^{b_1}_{b_3} \wedge b^{b_3}_{b_5}$$

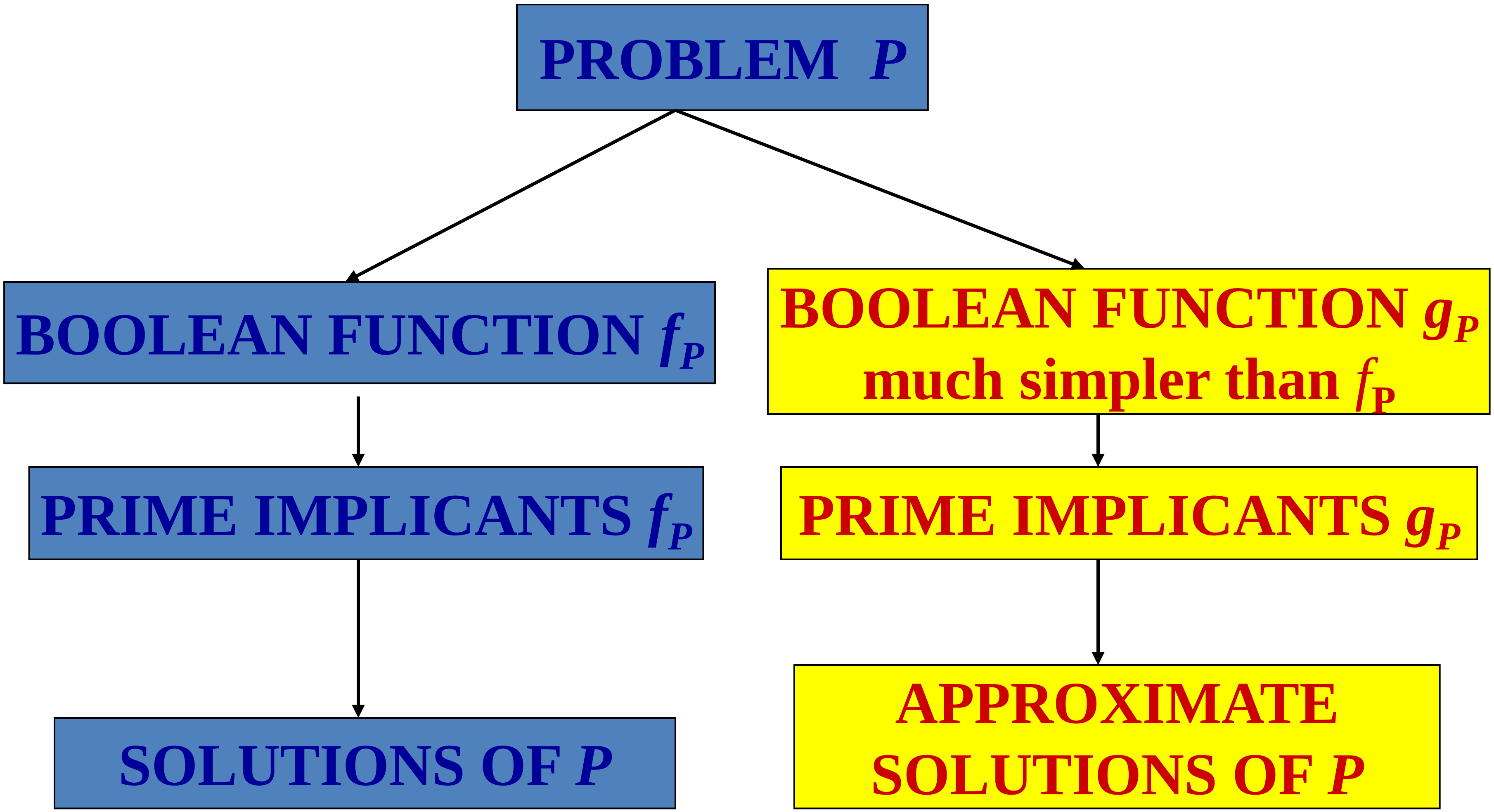




a^{P_a}	b^{P_b}	d
1	1	0
2	2	0
1	2	1
2	1	1

ASSOCIATION RULES
AND
α-REDUCTS

APPROXIMATE BOOLEAN REASONING



α-REDUCTS

For a given information system, $IS = (U,A)$ the set $B \subseteq A$ is called an **α-reduct**, if B has non-empty intersection with at least $\alpha \cdot 100\%$ of nonempty sets c_{ij} of the discernibility matrix of IS.

The problem of searching for *c*-irreducible **association rules** from a given template *T* of a decision table *DT* is equivalent to the problem of searching for local α-reducts of *DT*, for some α corresponding to *c*.

MORE APPLICATIONS

- Different criteria for discernibility: approximate reducts with respect to probability distribution, entropy reducts,... (D. Slezak, Ph.D. Thesis, Warsaw University)
- ...

DISCRETIZATION OF ATTRIBUTES OF DATA STORED IN RELATIONAL DATABASES

- Linear time for extracting partition with respect to number of cuts N is not acceptable because of time needed for one step (SQL query)
- Using approximate boolean reasoning based on simple statistics semi-optimal partition of high quality can be extracted in O(logN) time

Nguyen, Hung Son: On efficient handling of continuous attributes in large data bases, Fundamenta Informaticae 48(1) (2001), pp.61-81.

SCALABILITY

- **INFOBRIGHT**

- **USING SIMPLE STATISTICS OF DATA SETS FOR COMPUTING RELEVANT APPROXIMATE INFORMATION ABOUT DISCERNIBILITY (MATRICES) FUNCTIONS**

- **MapReduce + FPGA**

*www.**infobright**.com/*

Infobright's high-performance database

is the preferred choice for applications and data marts that analyze large volumes of "machine-generated data" such as Web data, network logs, telecom records, stock tick data and sensor data. Easy to implement and with unmatched data compression, operational simplicity and low cost, Infobright is being used by enterprises, … software companies in online businesses, telecommunications, financial services and other industries to provide rapid access to critical business data. For more information, please visit www.infobright.com or join our open source community at www.infobright.org.

NUMEROUS
RS APPLICATIONS
IN DIFFERENT DOMAINS

CURRENT RESEARCH DIRECTIONS AND CHALLENGES FOR COMBINATION OF BOOLEAN REASONING (AND OTHER APPROACHES) AND ROUGH SETS

- PATTERN RECOGNITION, MACHINE LEARNING, DATA MINING, DATA SCIENCE
- LARGE DATA SETS: EFFICIENT HEURISTICS FOR REDUCT GENERATION
- ADAPTIVE LEARNING
- CASE-BASED REASONING
- PLANNING
- HIERARCHICAL LEARNING
- ONTOLOGY APPROXIMATION
- SPATIO-TEMPORAL REASONING
- IoT, W2T, CYBER PHYSICAL SYSTEMS
- RS PROCESSOR (based on FPGA)
- ...

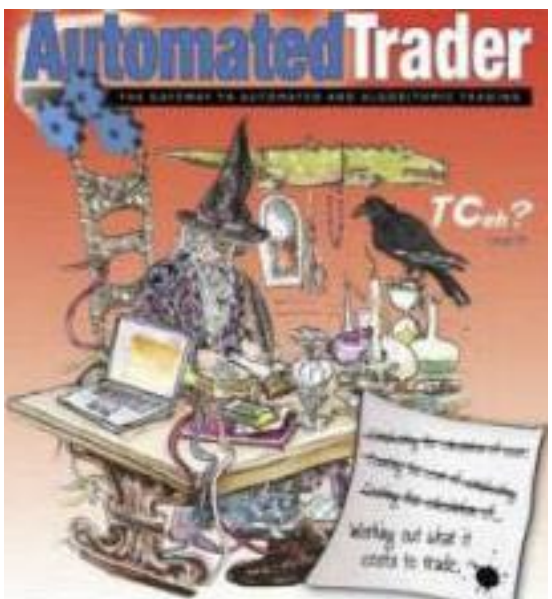
COMBINATIONS OF ROUGH SETS WITH OTHER APPROACHES

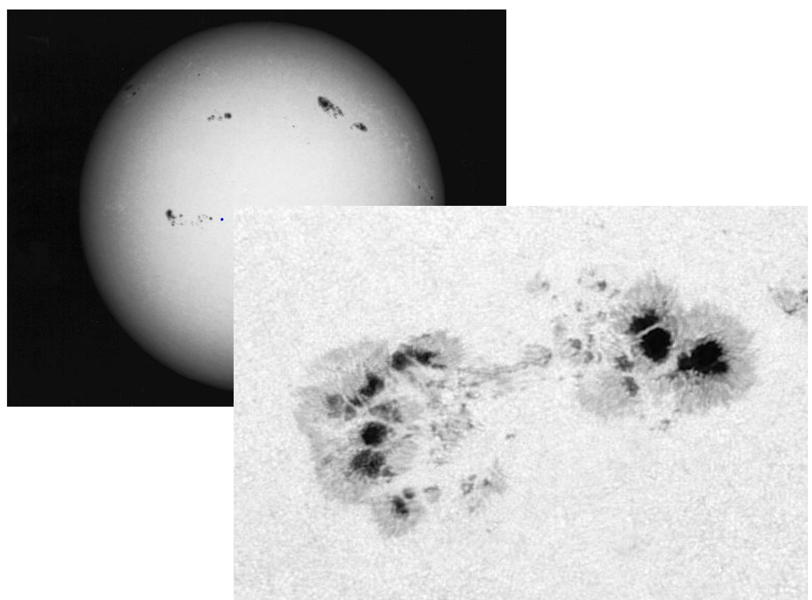
- FUZZY SETS
- NEURAL NETWORKS
- GENETIC ALGORITHMS AND EVOLUTIONARY PROGRAMMING
- STATISTICS
- GRANULAR COMPUTING
- WAVELETS, KERNEL FUNCTIONS, CASE-BASED REASONING, EM METHOD, INDEPENDENT COMPONENT ANALYSIS, PRINCIPAL COMPONENT ANALYSIS
- ...

ROUGH SETS AND APPROXIMATION OF COMPLEX VAGUE CONCEPTS : ONTOLOGY APPROXIMATION

- Making progress in constructing of the high quality intelligent systems
- Examples: approximation of complex vague concepts such as guards of actions or behavioral patterns
- Reasoning about vague concepts

APPLICATIONS :
APROXIMATION OF COMPLEX VAGUE CONCEPTS





REAL-LIFE PROJECTS

UAV control of unmaned helicopter (Wallenberg Foundation, Linkoeping University)

Medical decision support (glaucoma attacs, respiratory failure,…)

Fraud detection (Bank of America)

Logistics (Ford GM)

Dialog Based Search Engine (UNCC, Excavio)

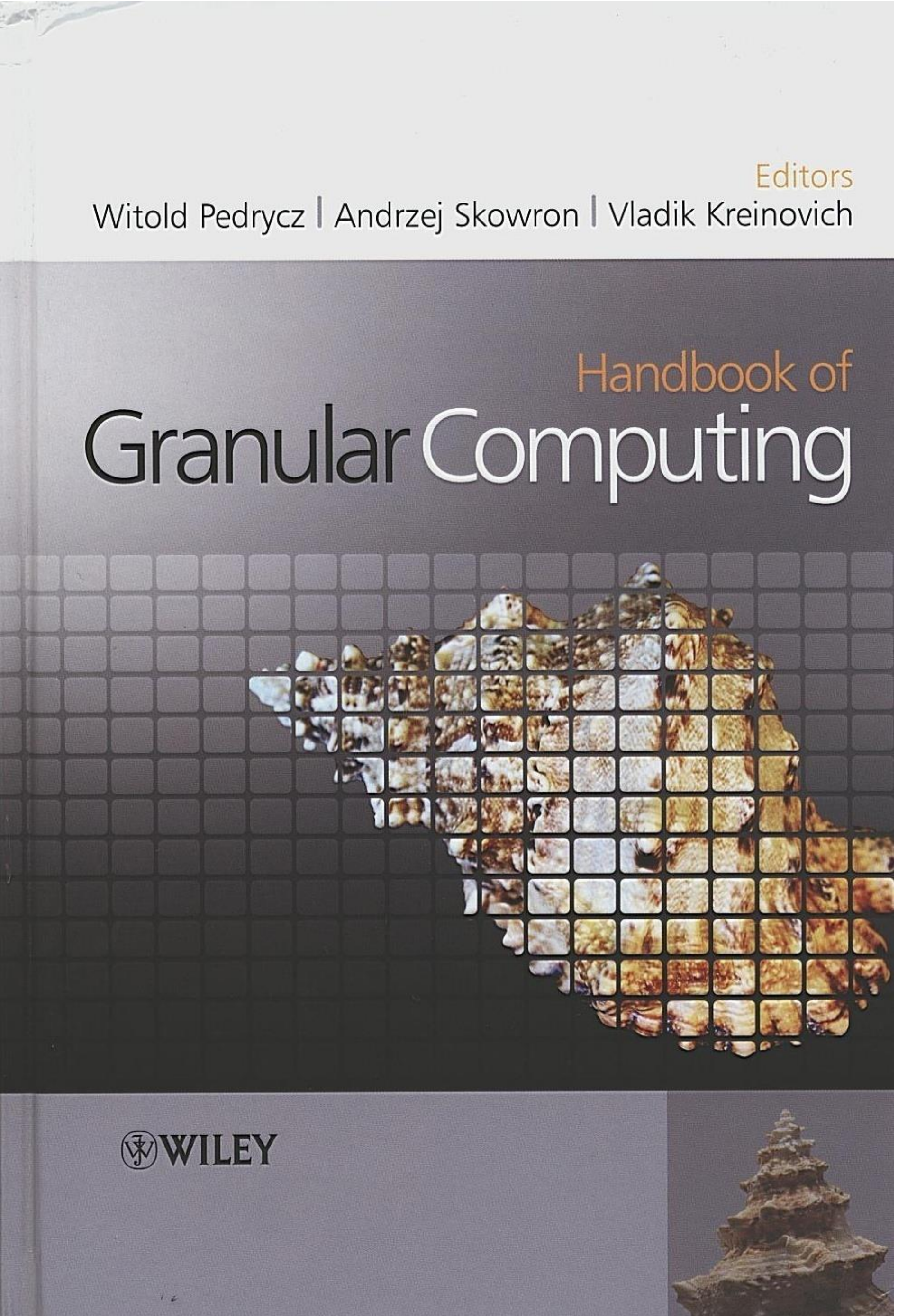
Algorithmic trading (Adgam)

Semantic Search (SYNAT) (NCBiR)

Firefighter Safty (NCBiR)

…

ROUGH SETS (RS)
AND
GRANULAR COMPUTING (GC)



Plays a key role in implementation of the strategy of divide-and-conquer in human problem-solving – Lotfi Zadeh

Zadeh, L. A. (1979) Fuzzy sets and information granularity. In: Gupta, M., Ragade, R., Yager, R. (eds.), Advances in Fuzzy Set Theory and Applications, Amsterdam: North-Holland Publishing Co., 3-18

Zadeh, L.A. (2001) A new direction in AI-toward a computational theory of perceptions. AI Magazine 22(1): 73-84

LESLIE VALIANT: TURING AWARD 2010

March 10, 2011:

Leslie Valiant, of Harvard University, has been named the winner of the 2010 Turing Award for his efforts to develop computational learning theory.

http://www.techeye.net/software/leslie-valiant-gets-turing-award#ixzz1HVBeZWQL

Current research of Professor Valiant

http://people.seas.harvard.edu/~valiant/researchinterests.htm

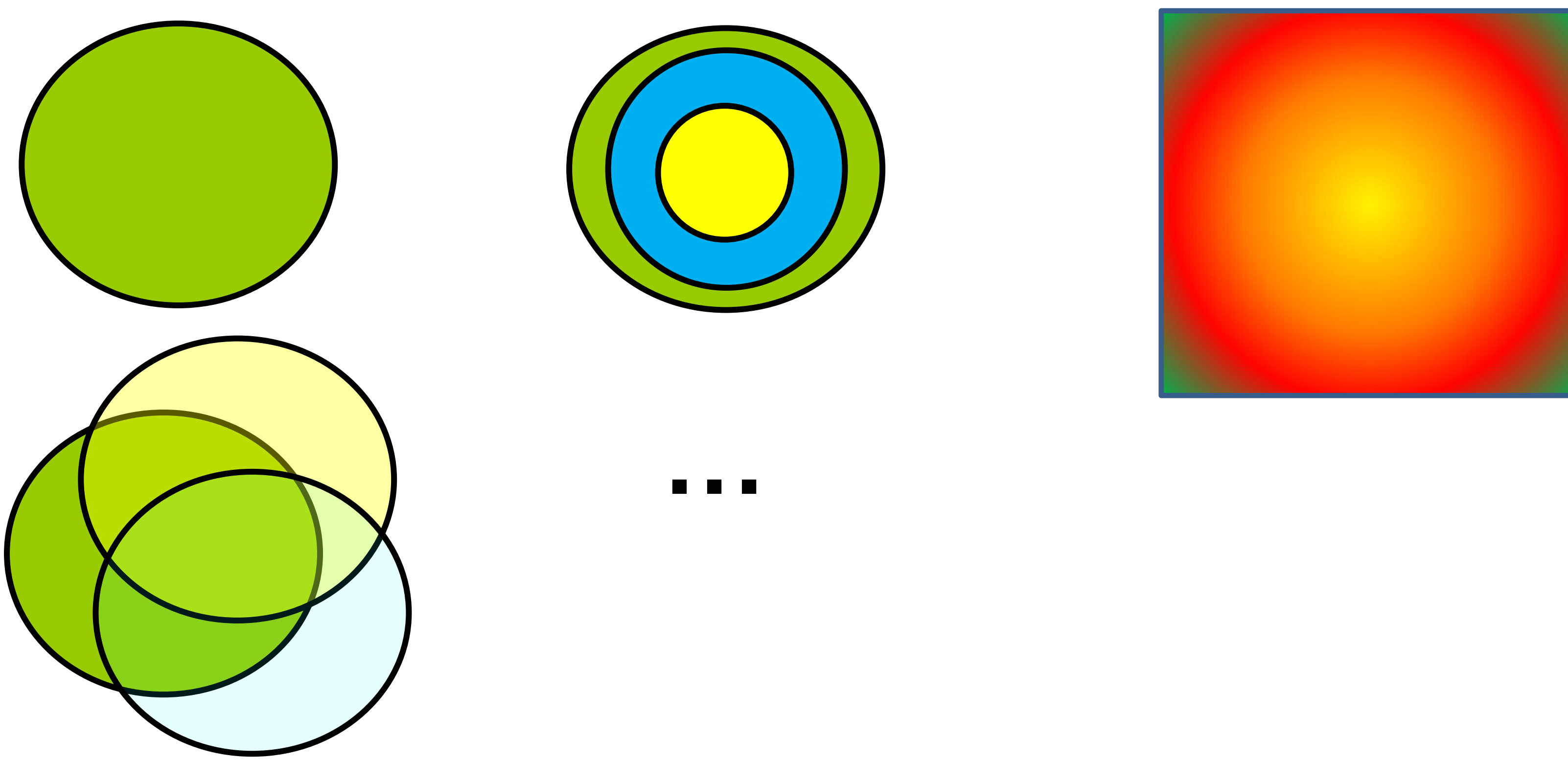
A fundamental question for artificial intelligence is to characterize the computational building blocks that are necessary for cognition.

INFORMATION GRANULES

ELEMENTARY GRANULES

+

INTERACTIVE CALCULULI OF GRANULES



DEFINABLE GRANULES

ROUGH GRANULES

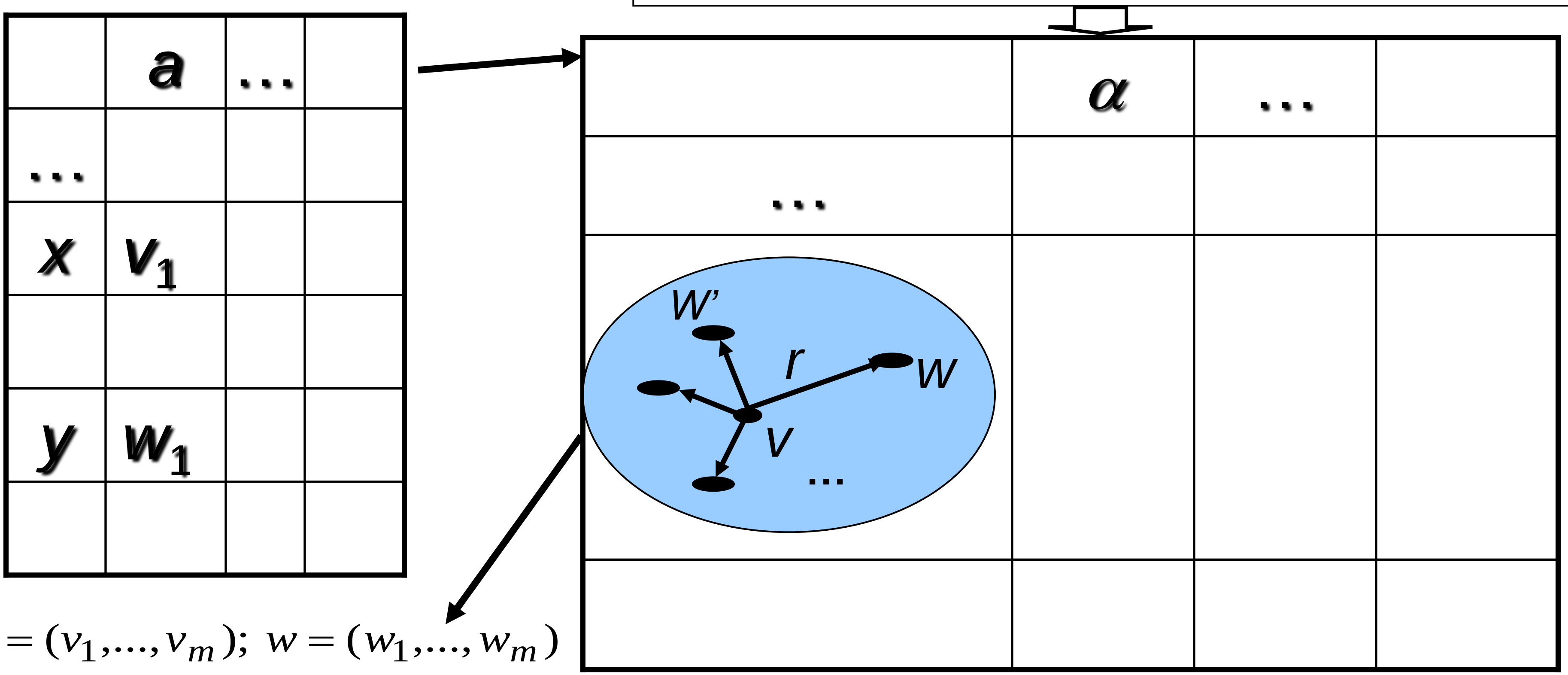
APPROXIMATION OF GRANULES

STRUCTURAL OBJECTS

SEARCHING FOR RELEVANT FEATURES

GENERALIZATIONS OF GRANULES: TOLERANCE GRANULES

invariants over tolerance classes; compare invariants in the Gibson approach



$$v = (v_1, \ldots, v_m);\ w = (w_1, \ldots, w_m)$$

$$v\, r\, w \ \mathit{iff}\ \ v_i r_i w_i \ \ \mathit{for}\, i = 1, \ldots m$$

$$r(v) = \{w : v\, r\, w\}$$

$$\|r(v)\| = \bigcup\{\|w\| : w \in r(v)\}$$

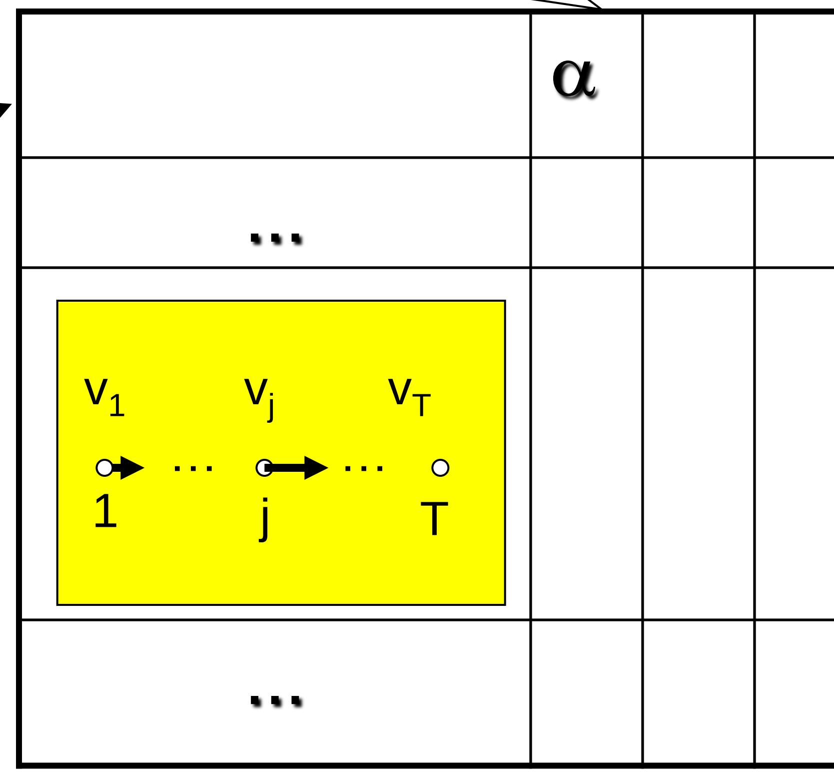
GENERALIZATION

from *v* to *r*(*v*)

GRANULES REPRESENTING STRUCTURES OF OBJECTS

properties of time windows

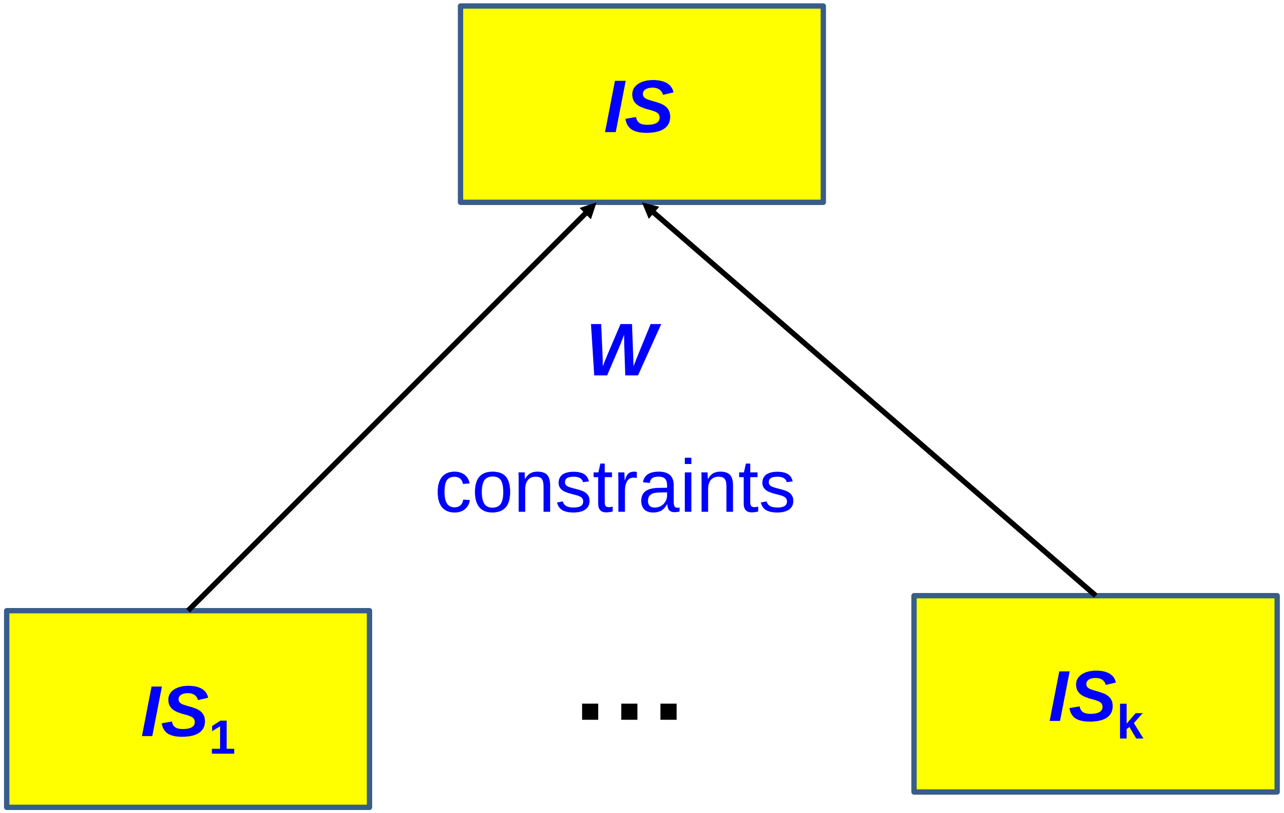
TIME WINDOWS



	t	t_T	a_1	...
...	...	...		
x	i	mod(i,T)	$V_{1,i}$	...
...	...			

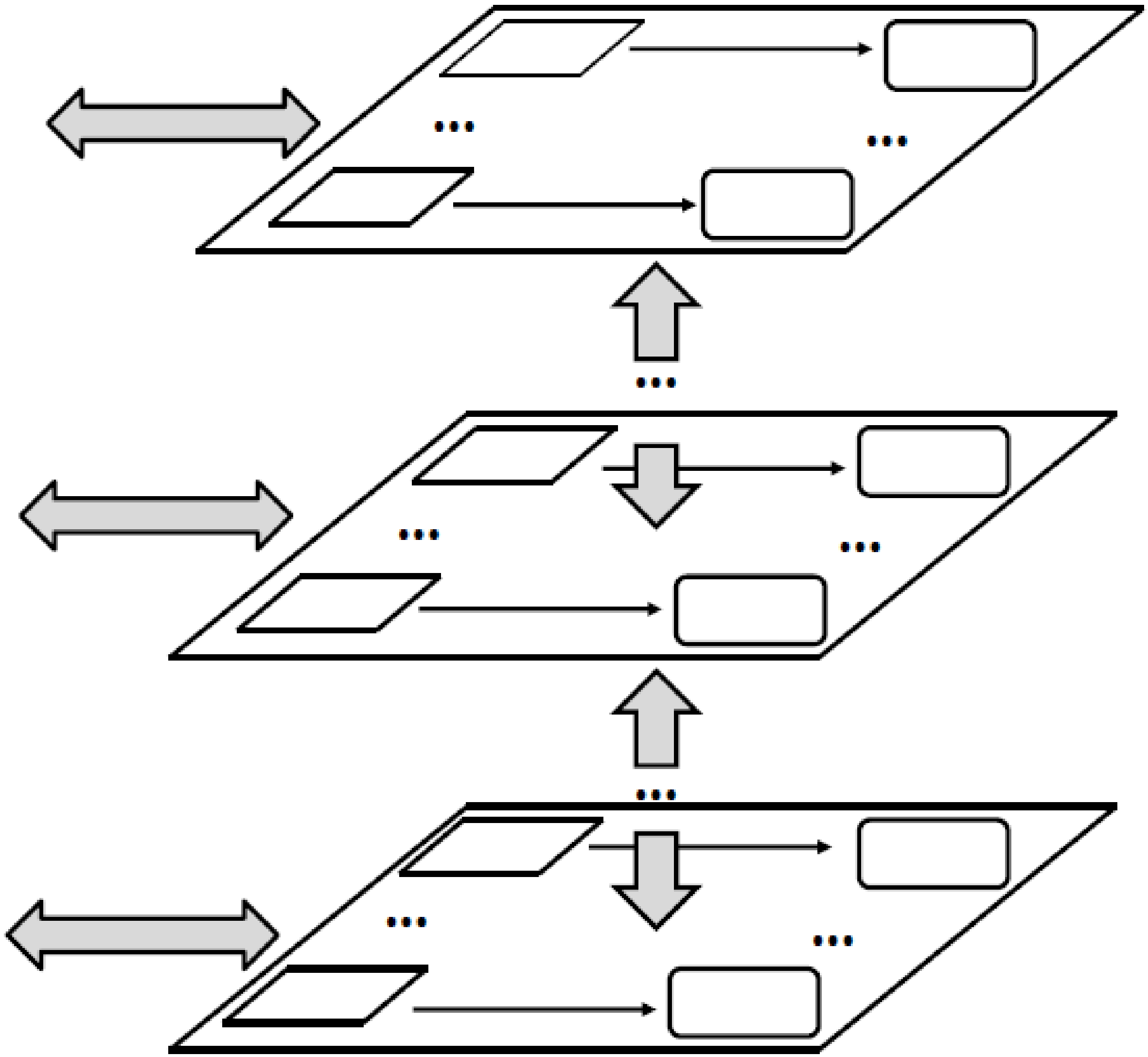
$V_i=(v_{1,i},\ldots,v_{m,i})$

JOIN WITH CONSTRAINTS

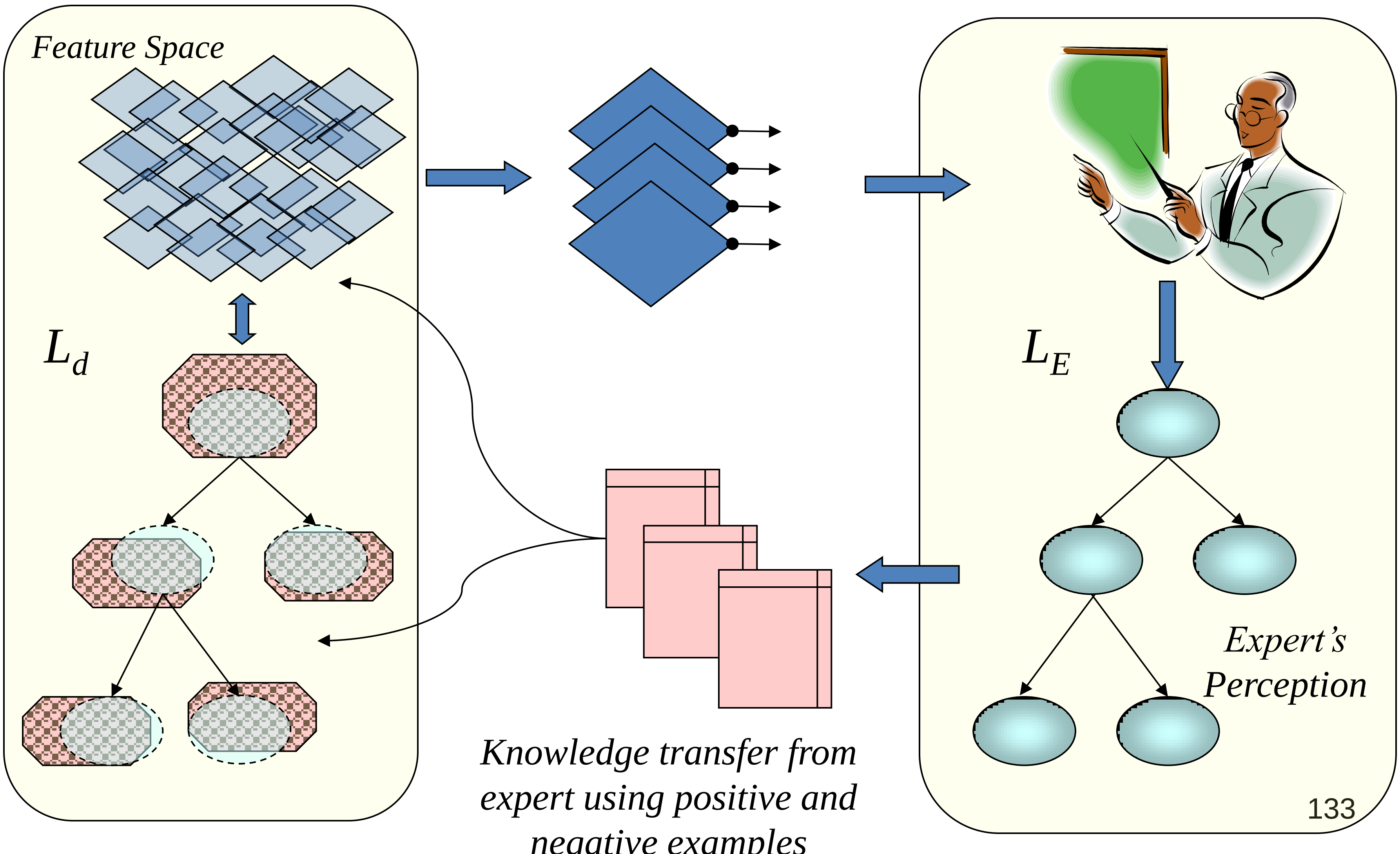


Objects (granules) in *IS* are composed out of attribute value vectors from $IS_1 \ldots IS_k$ satisfying *W*

INTERACTIVE HIERARCHICAL STRUCTURES



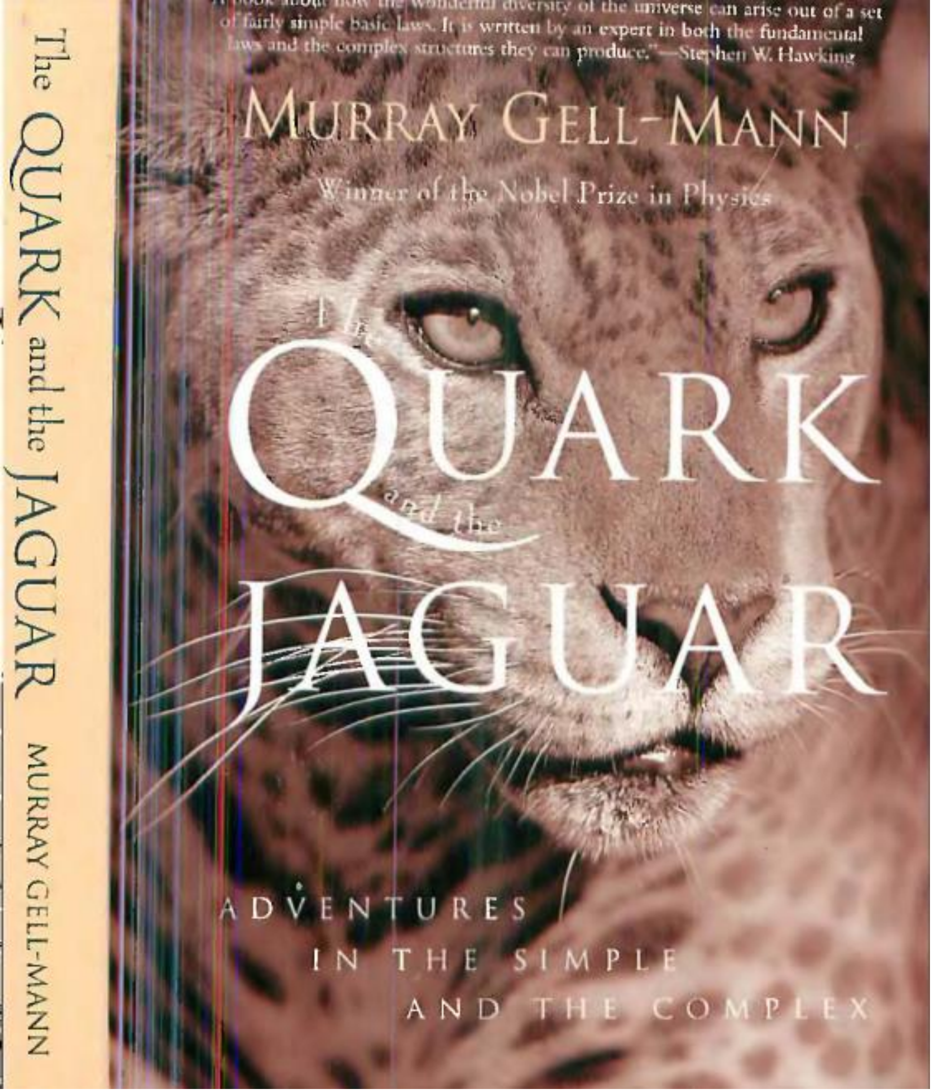
ROUGH SET BASED ONTOLOGY APPROXIMATION



WHAT NEXT?

COMPLEX ADAPTIVE SYSTEMS (CAS):

USING BIG DATA GENERATED BY CAS FOR ANALYSIS AND SYNTHESIS (INCLUDING CONTROL)



The jaguar stands for the complexity of the world around us, especially as manifested in complex adaptive systems.

In much of today's research on complex adaptive systems, mathematics plays a very significant role, but in most cases it is not the kind of mathematics that has traditionally predominated in scientific theory.

IN DEALING WITH
COMPLEX SYSTEMS:

MORE COMPLEX VAGUE CONCEPTS
SHOULD BE APPROXIMATED
AND
NEW KIND OF REASONING ABOUT
COMPUTATIONS PROGRESSING
BY ITERACTIONS AMONG
LINKED MENTAL AND/OR PHYSICAL
OBJECTS IS NEEDED

COMPLEX (ADAPTIVE) SYSTEMS

Etymologically: complexity – plexus in Latin (interwoven). Complex system: the elements are difficult to separate. **This difficulty arises from the interactions between elements**. **Without interactions, elements can be separated.**

But **when interactions are relevant, elements co-determine their future states. Thus, the future state of an element cannot be determined in isolation, as it co-depends on the states of other elements, precisely of those interacting with it.**

Gershenson, C. and Heylighen, F. (2005). How can we think the complex? In Managing Organizational Complexity: Philosophy, Theory and Application, K. Richardson, (Ed.). Information Age Publishing, Chapter pp. 47-61.

COMPLEX ADAPTIVE SYSTEMS (CAS)

- Exhibiting internal boundaries dividing any of such system into a diverse array of semi-autonomous subsystems called agents; agent has a ``program'' guiding its interactions with other agents and other parts of its environment.
- CAS are signal/boundary systems.The steering of CAS is expressed by modifying signal/boundary hierarchies.
- **Interactions are basic concepts of the approach.** Categories of interactions in signal/boundary systems: diversity, recirculation, niche, and coevolution.

John Holland: Signals and Boundaries. Building Blocks for Complex Adaptive Systems MIT Press 2012.

COMPLEX (ADAPTIVE) SYSTEMS

We can find examples of complex systems all around us :

- cells are composed of interacting molecules,
- brains are composed of interacting neurons,
- societies are composed of interacting individuals,
- ecosystems are composed of interacting species.

MORE EXAMPLES OF COMPLEX SYSTEMS

SOFTWARE PROJECTS

MEDICAL SYSTEMS

ALGORITHMIC TRADING

SYSTEMS INTEGRATING TEAMS OF ROBOTS AND HUMANS

TRAFFIC CONTROL SYSTEMS

PERCEPTION BASED SYSTEMS

ULTRA LARGE SCALE YSTEMS

...

CYBER-PHYSICAL SYSTEMS

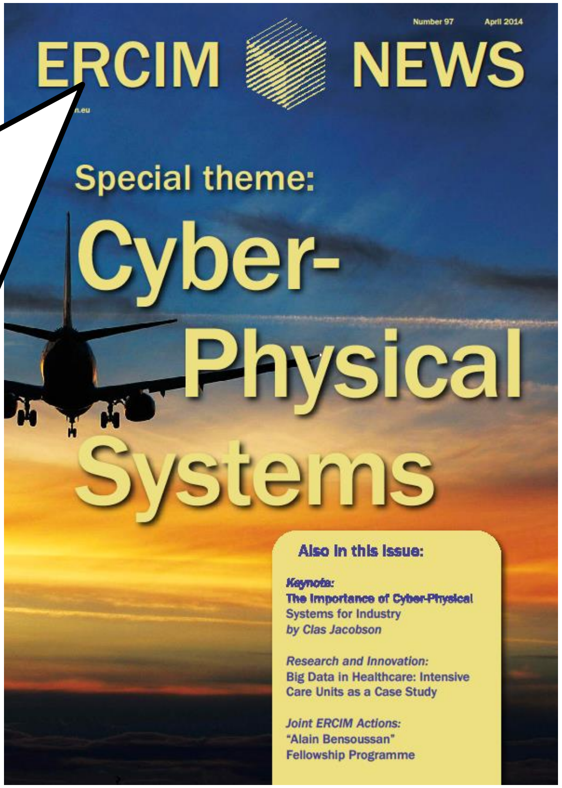
A **cyber-physical system** (**CPS**) is a system of collaborating computational elements controlling physical entities.

Cyber-Physical Systems will transform **how we interact with the physical world** just as the Internet transformed how we interact with one another.

Applications with enormous societal impact and economic benefit will be created.

CYBER-PHYSICAL SYSTEMS (CPS)

European Research Consortium for Informatics and Mathematics



Smart Medical Technologies: e.g., Personal Heart Monitoring System Using Smart Phones To Detect Life Threatening Arrhythmias

Firefighting, e.g. on-line decision support for fire commander

Coordination (e.g., air traffic control, road traffic control)

Autonomous Vehicles and Smart Transportation

Smart cities

Security

CYBER-PHYSICAL SYSTEMS

… the size of cyber-physical systems of systems and their ‘multimodality’ or hybrid nature consisting of physical elements as well as quasicontinuous and discrete controls, communication channels, and local and system-wide optimization algorithms and management systems, implies that hierarchical and multi-domain approaches to their simulation, analysis and design are needed. **These methods are currently not available.**

WISDOM WEB OF THINGS (W2T)

[Hyper world] consists of the cyber, social, and physical worlds, and uses data as a bridge to connect humans, computers, and things. ... [Wisdom Web of Things] W2T focuses on the data cycle, namely "from things to data, information, knowledge, wisdom, services, humans, and then back to things." A W2T data cycle system is designed to implement such a cycle, which is, technologically speaking, a practical way to realize the harmonious symbiosis of humans, computers, and things in the emerging hyper world.

N. Zhong, J.H. Ma, R.H. Huang, J.M. Liu, Y.Y. Yao, Y.X. Zhang, and J.H. Chen: Research Challenges and Perspectives on Wisdom Web of Things (W2T). Journal of Supercomputing, Springer, Volume 64(3) (2013) 862-882.

GAP BETWEEN THEORY AND PRACTICE

Human Interaction, Computational Emergence, Design, Computational Engineering, Adaptive System Infrastructure, Adaptable and Predictable System Quality, Policy, Acquisition, and Management, ...

Progress has been made on all these fronts and others.

And yet ... there is a fast growing gap between our research and reality.

Linda Northrop1: Does Scale Really Matter?: Ultra-Large-Scale Systems Seven Years after the Study. Software Engineering Institute, Carnegie Mellon University (2013)

PROBLEMS:

MODELS OF INTERACTIVE COMPUTATIONS

COMPARISON WITH TURING MODEL

CHALLENGES FOR LOGIC AND RS

STRATEGIES FOR ADAPTIVE LEARNING
INTERACTIVE RULES FOR CONTROL

INTERACTIONS

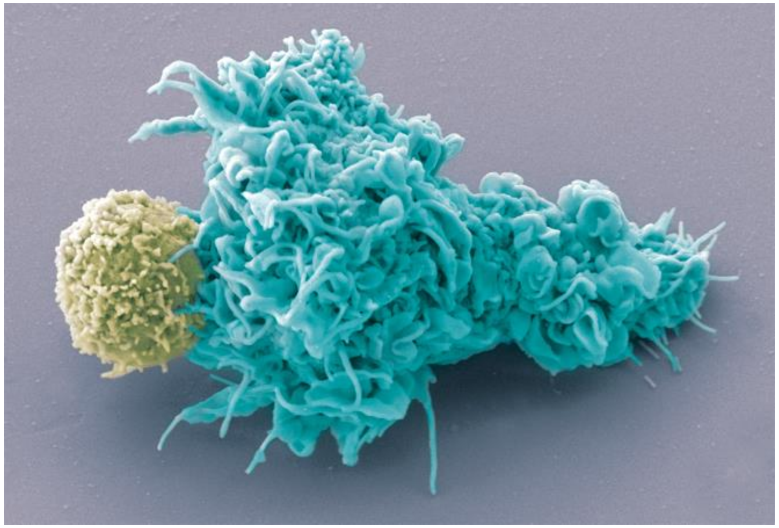
INTERACTIONS

[…] **interaction** is a critical issue in the understanding of complex systems of any sorts: as such, it has emerged in several well-established scientific areas other than computer science, like biology, physics, social and organizational sciences.

Andrea Omicini, Alessandro Ricci, and Mirko Viroli, The Multidisciplinary Patterns of Interaction from Sciences to Computer Science. In: D. Goldin, S. Smolka, P. Wagner (eds.): Interactive computation: The new paradigm, Springer 2006

INTERACTIONS

[…] One of the fascinating goals of natural computing is to understand, in terms of information processing, the functioning of a living cell. An important step in this direction is understanding of **interactions** between biochemical reactions. … the functioning of a living cell is determined by **interactions** of a huge number of biochemical reactions that take place in living cells.



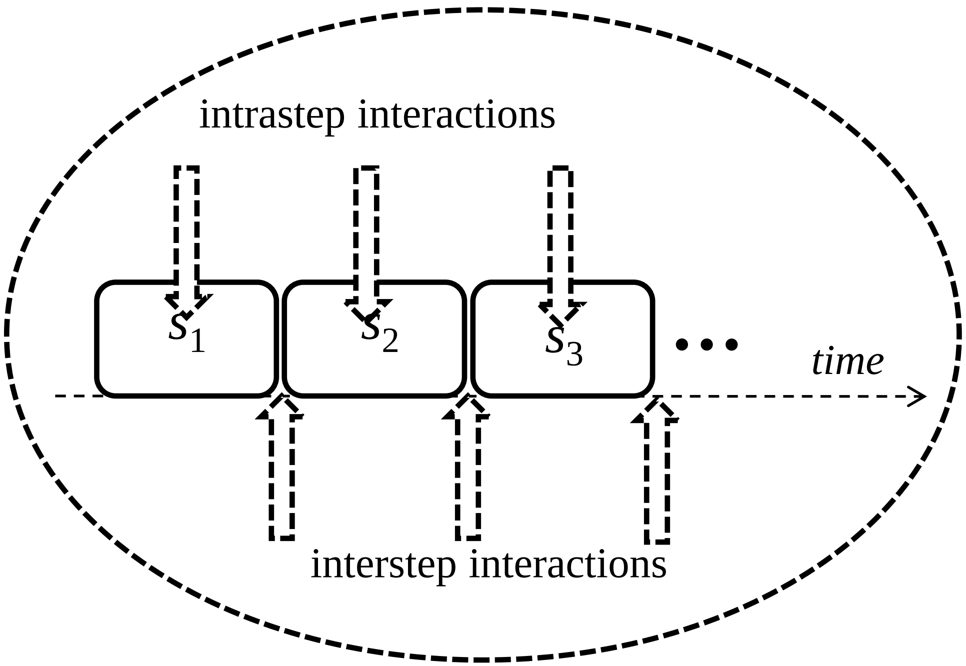
A human dendritic cell (blue pseudo-color) in close interaction with a lymphocyte (yellow pseudo-color). This contact may lead to the creation of an immunological synapse.

The Immune Synapse by Olivier Schwartz and the Electron Microscopy Core Facility, Institut Pasteur

http://www.cell.com/Cell_Picture_Show

Andrzej Ehrenfeucht, Grzegorz Rozenberg: Reaction Systems: A Model of Computation Inspired by Biochemistry, LNCS 6224, 1–3, 2010

INTERSTEP vs INTRASTEP INTERACTIONS

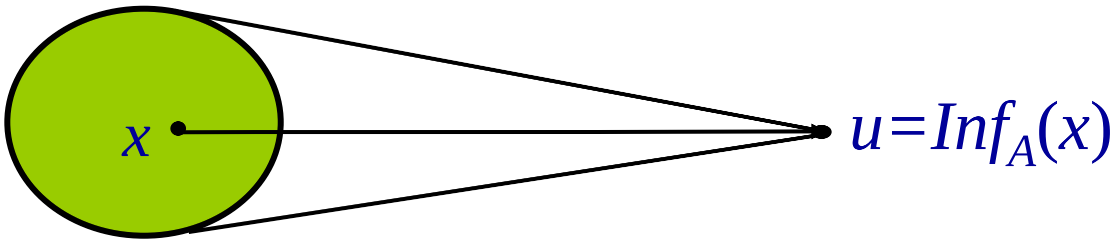


Gurevich, Y.: Interactive Algorithms . In: D. Goldin, S. Smolka, P. Wagner (eds.): Interactive computation: The new paradigm, Springer 2006

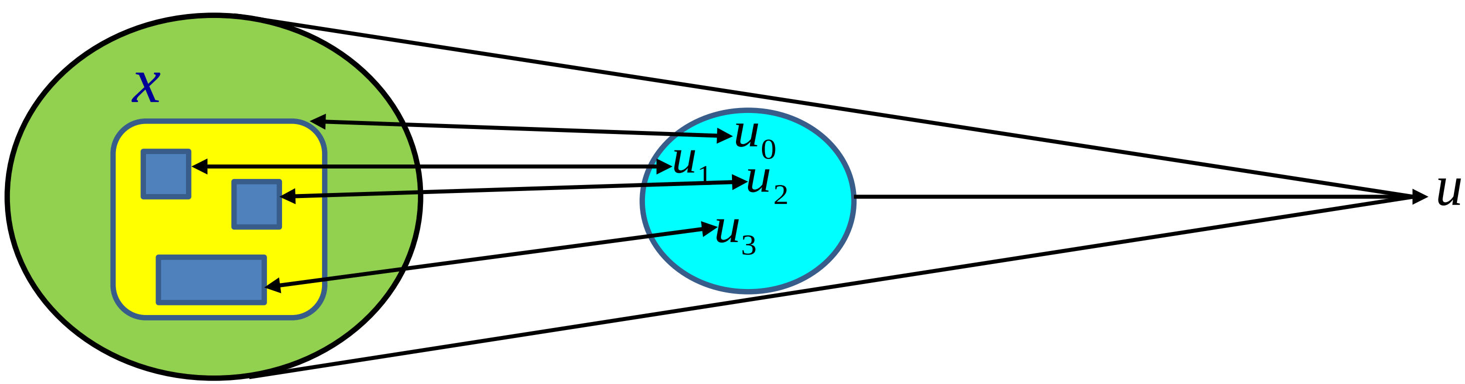
FROM GC TO INTERACTIVE GC

COMPUTATIONS BASED ON INTERACTIONS OF COMPLEX GRANULES

INTERACTIVE INFORMATION SYSTEMS



for complex physical objects we need to model interaction with them



SYMBOL GROUNDING PROBLEM

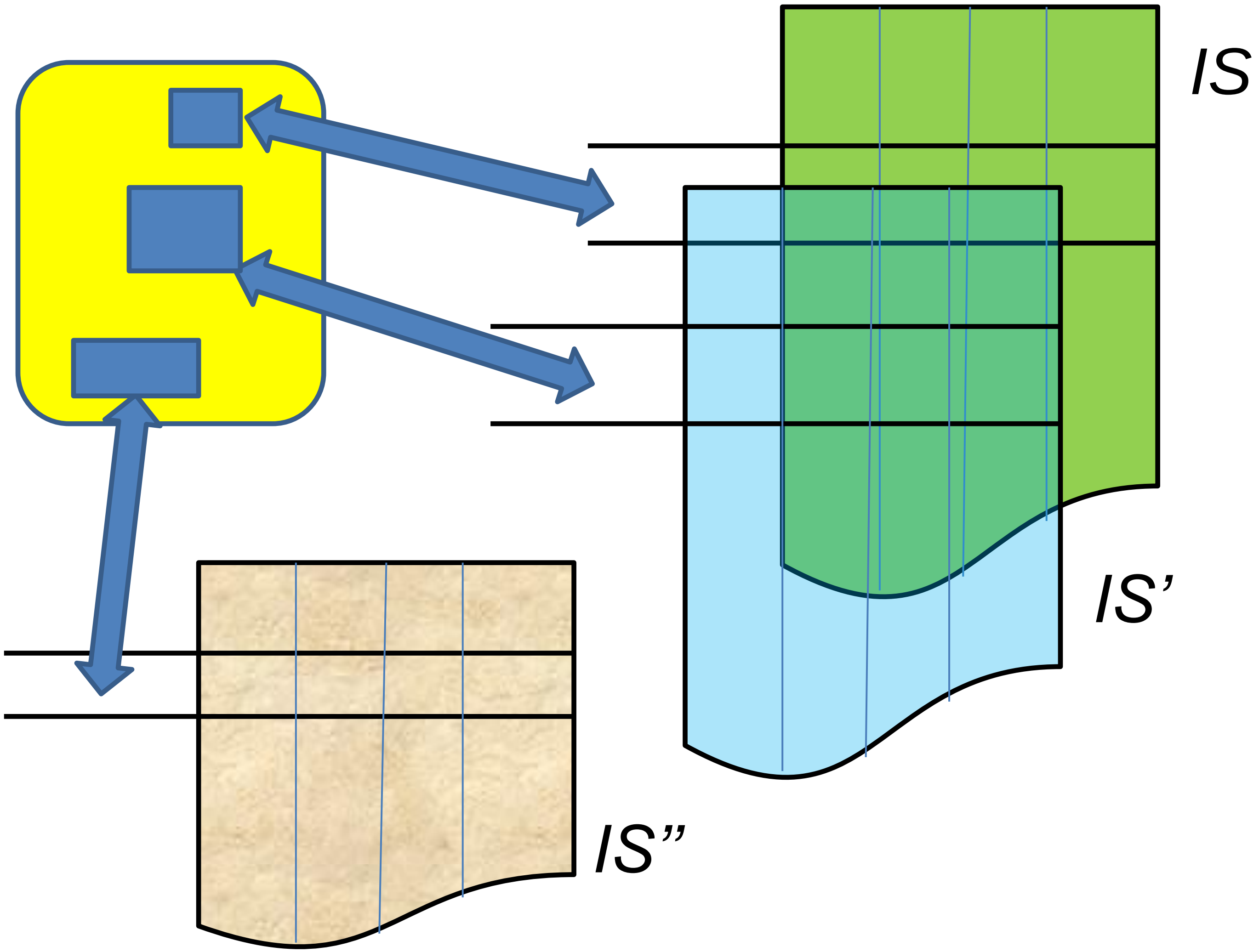
Stevan Harnad: Symbol grounding problem.
Physica D 42: 335-346, 1990

A long-standing concern when constructing models of cognitive systems is how to characterize the relationship between the states inside the system, and the objects in the external world that they purportedly represent.

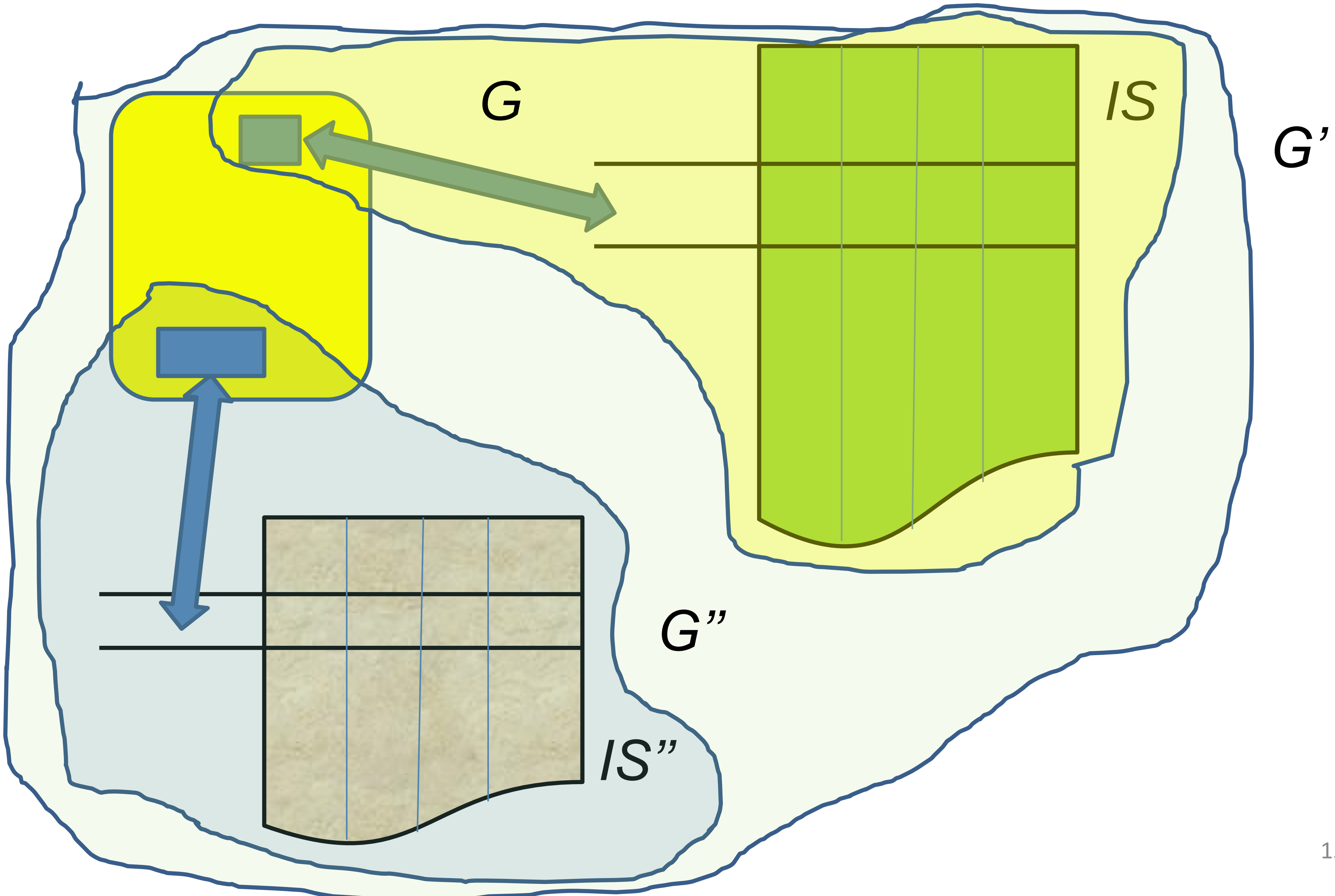
SEMANTIC POINTERS

Chris Eliasmith: How to build a brain.
Oxford University Press, 2013

INTERACTIVE INFORMATION SYSTEMS ARE LINKED WITH PHYSICAL OBJECTS BY COMPLEX GRANULES (c-granules)



INTERACTIVE INFORMATION SYSTEMS ARE LINKED WITH PHYSICAL OBJECTS BY COMPLEX GRANULES (c-granules)



C-GRANULE : INTUITION

C-granules generated by *ag* are configurations linked by *ag* in a special way using hunks. The control of an agent *ag* is using her/his c-granules for accessing fragments of the surrounding her/him physical world. Any c-granule consists of three layers:

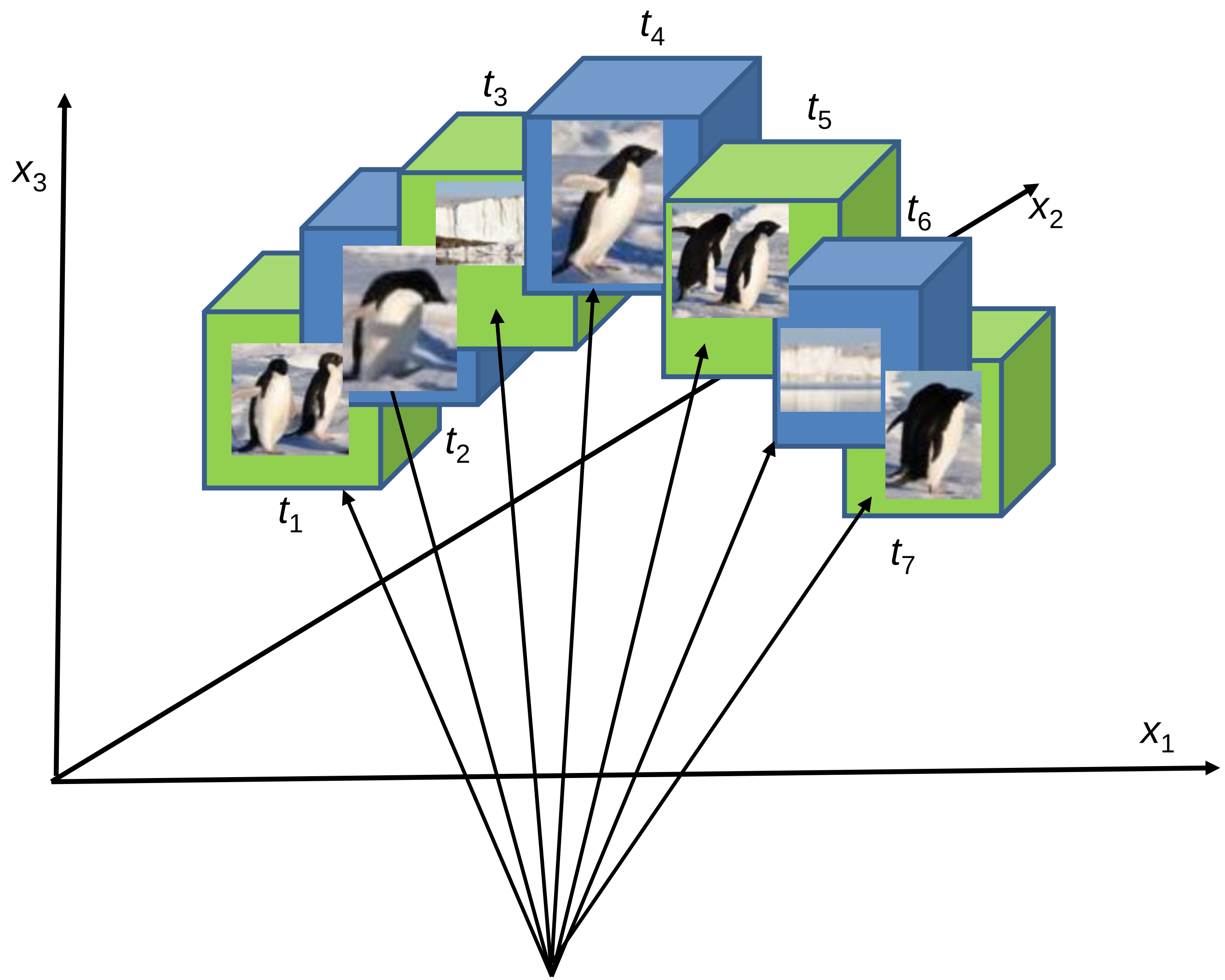
1. soft_suit , i.e., configurations of hunks representing properties of the *ag* activity environment (among them properties of present, past, and expected phenomena as well as expected properties of results of some interactions potentially activated by the c-granule);
2. link_suit , i.e., communication channels (links) transmitting results of interactions among accessible fragments of the *ag* activity environment and results of interactions among representations of properties in the soft_suite; priorities mab be assigned to links reflecting the results of judgement by *ag* of their weights relative to the current needs hierarchy of *ag*;
3. hard_suit, i.e., are configurations of hunks accessible by links from link_suit.

C-GRANULE : INTUITION

C-granules of *ag* support such activities of *ag* as

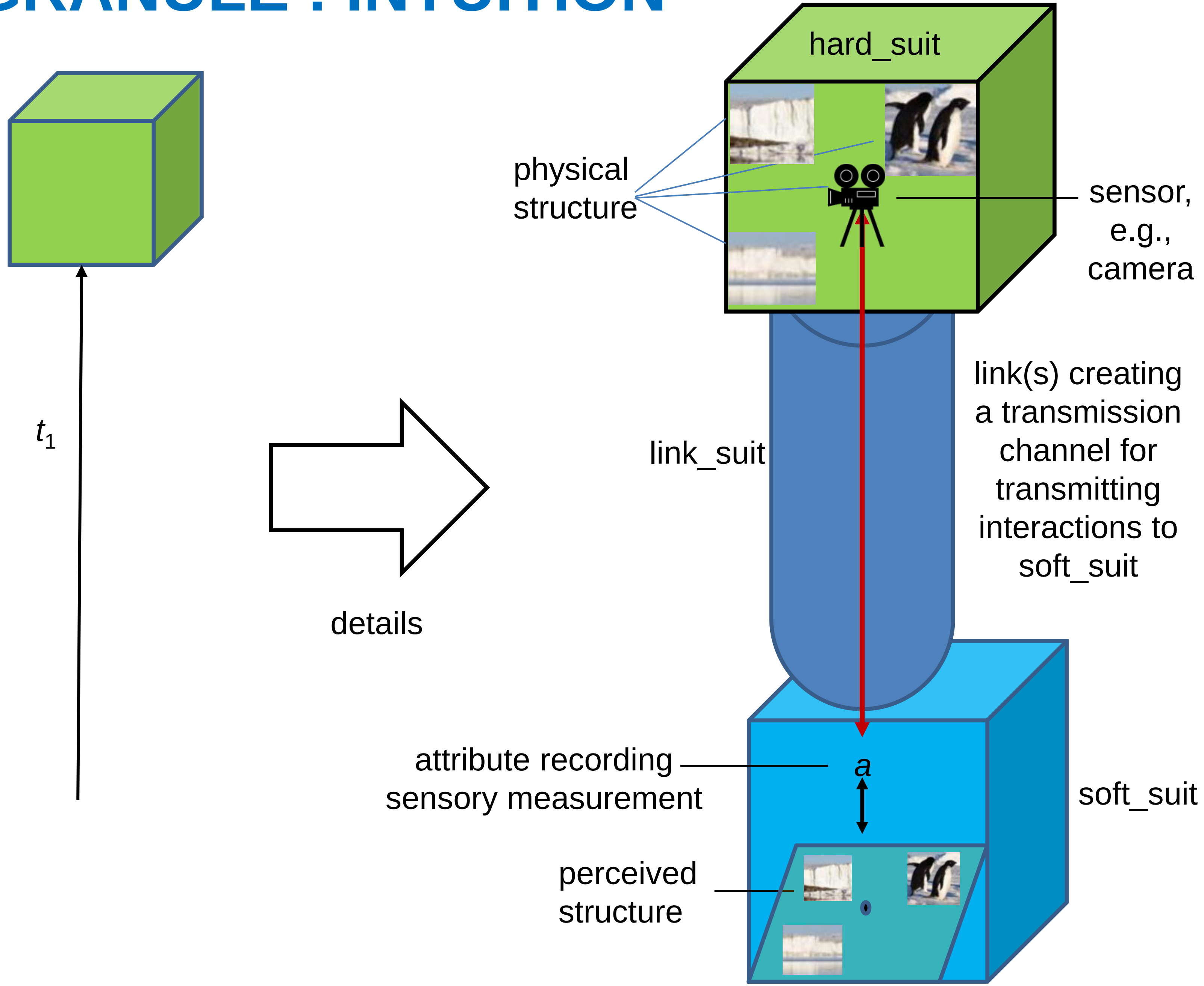
1. improving by *ag* representation techniques of her/his hierarchy of needs and her/his techniques of perception of needs as well as relations between them;
2. interpretation and judgement by ag of importance of phenomena taking place in her/his activity environment;
3. judgement by *ag* of phenomena in her/his environment (in particular, of causes and consequences of the phenomena from the perspective of her/his hierarchy of needs;
4. construction, initialization, realization, verification, adaptation, and termination of interaction plans by *ag*;
5. communication, cooperation and competition of *ag* with other agents.

FROM HUNK TO C_GRANULE: INTUITION

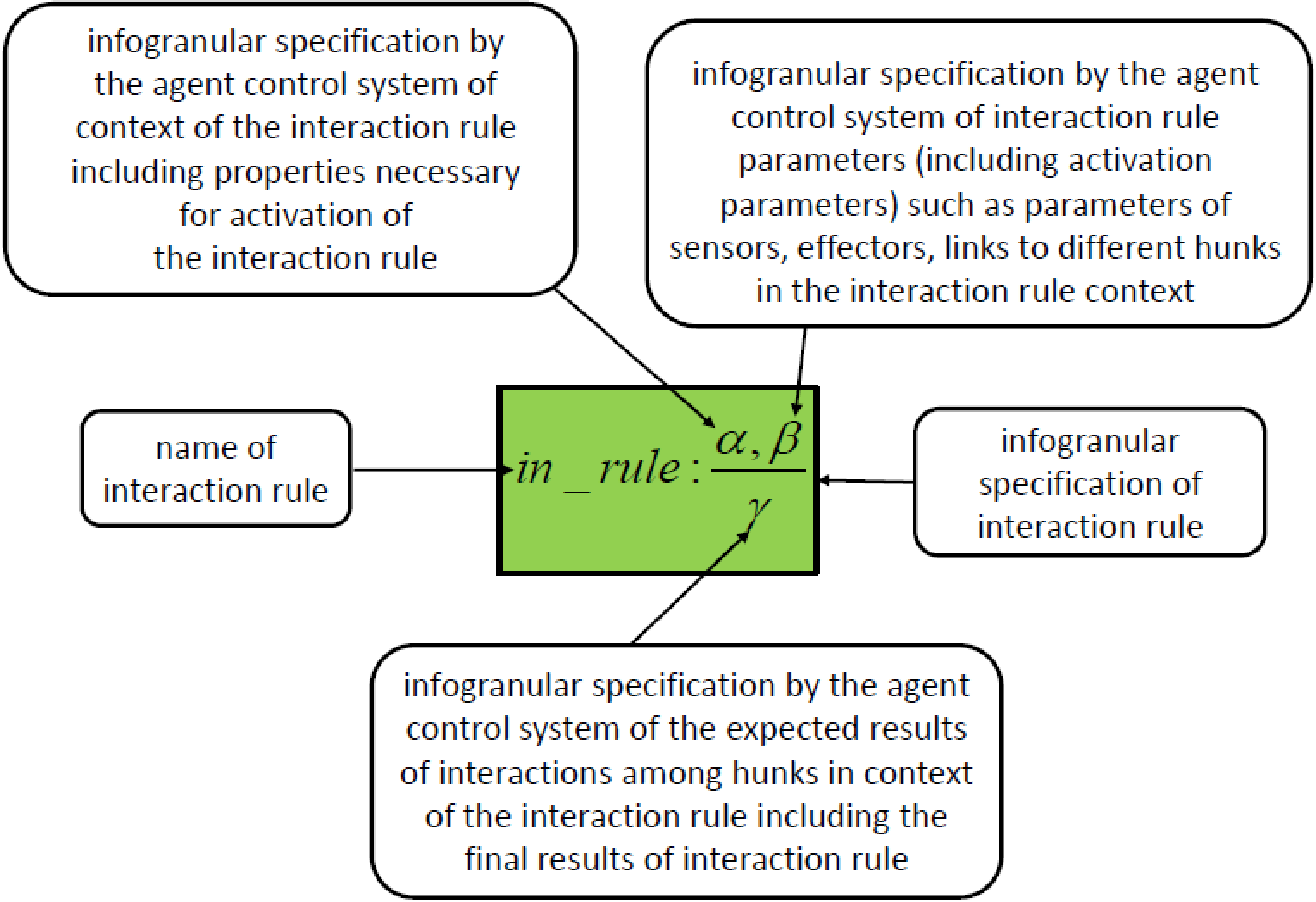


Links to a spatiotemporal hunk specified by the agent control using 'windows', pointing to different fragments (portions of matter) of the in 3 dimensional physical world in different moments (or periods) of time t_1 ,…, t_7

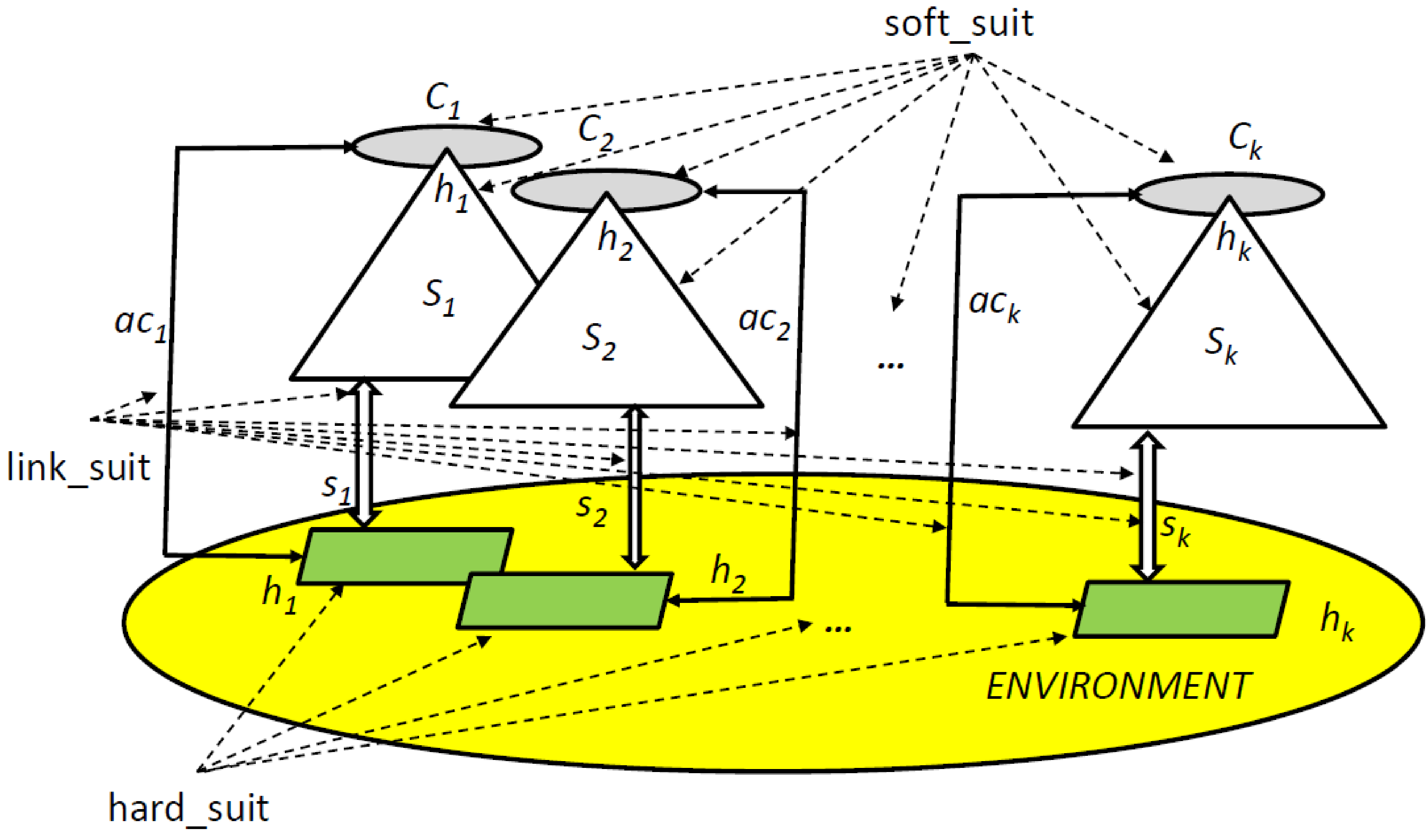
C-GRANULE : INTUITION



INTERACTION RULE



COMPLEX GRANULES



LESLIE VALIANT: TURING AWARD 2010

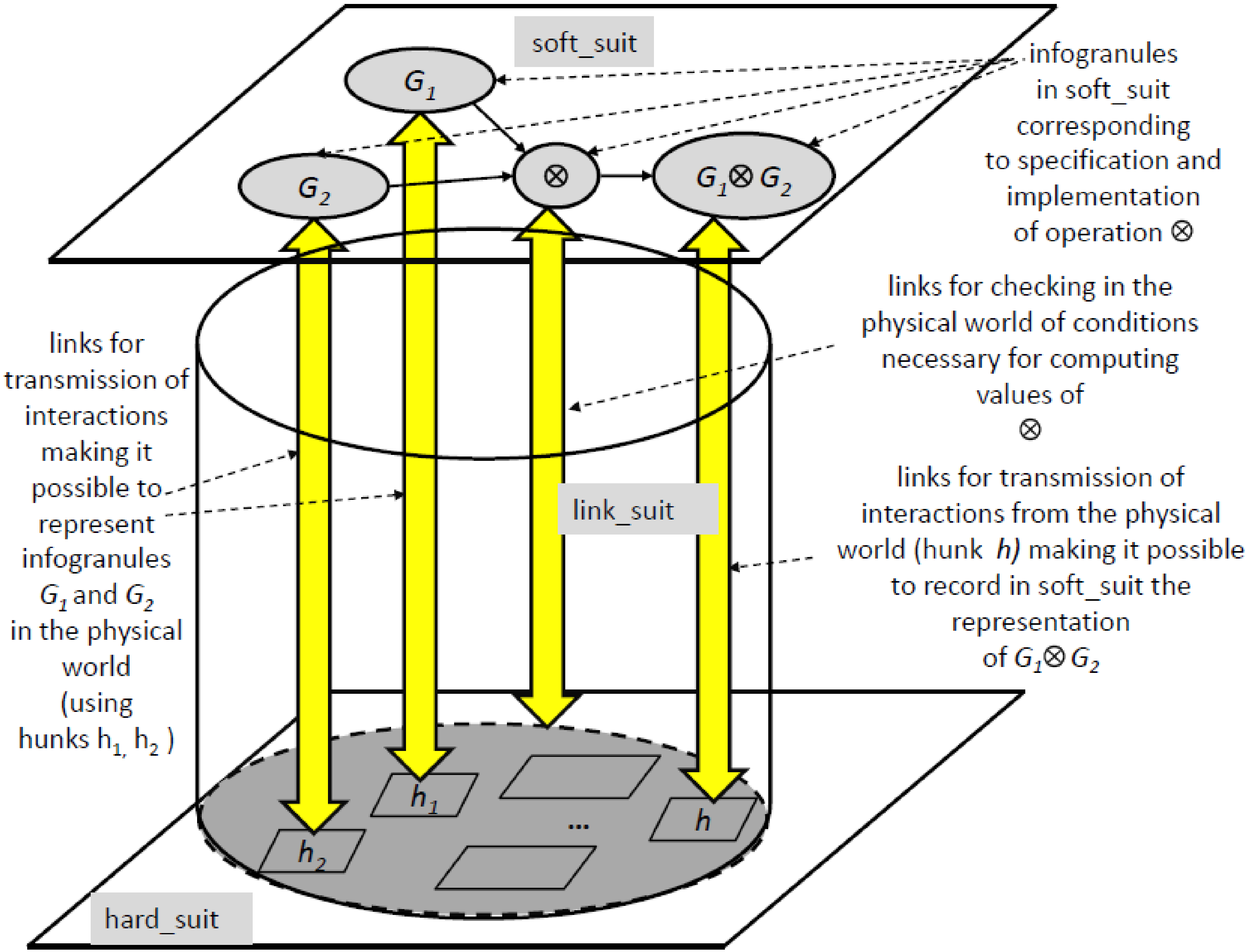
.

A specific challenge is to build on the success of machine learning so as to cover broader issues in intelligence.

This requires, in particular a reconciliation between two contradictory characteristics -- the apparent logical nature of reasoning and the statistical nature of learning.

Professor Valiant has developed a formal system, called robust logics, that aims to achieve such a reconciliation.

INTERACTIVE COMPUTABILITY vs TURING COMPUTABILITY



THE CASE FOR QUANTUM COMPUTING
Andrew Yao (WIC 2014 Panel)

A Disruptive computing paradigm:

- Compute f(x) by a *gedanken* experiment:
  1. **Grow a crystal C tailored for f, x**
  2. **Shine an optical wave on C**
  3. **From the diffraction pattern, figure out f(x)**

- Magic of quantum software simulation:
  exponentially speedup over classical hardware

INTERACTIVE COMPUTABILITY
vs
TURING COMPUTABILITY

The operations of aggregation of c-granules are computationally admissible, only if we can realize them in the physical world.

Computations on c-granules run in environments unknown to the agent, and they are allowing for learning by interacting with the environment how to act effectively in it. After sufficient interaction they lead to the agent expertise not provided by her/him, but extracted from the environment.

COMPLEXITY OF ONTOLOGY

Ontologies of agents or their societies are complex. One can understand better the complexity of such task referring to the research on the cognitive systems such as

SOAR or ACT-R

aiming at constructing a general cognitive architecture for developing systems that exhibit intelligent behavior. The research on development of such systems, initiated many years ago, is still very active and carried out by different groups of researchers.

NICHES

J.Holland: Signals and Boundaries.
Building Blocks for Complex Adaptive Systems, MIT 2012

A niche is a diverse array of agents that regularly exchange resources and depend on that exchange for continued existence. […] The niche, then, is made up of physical and virtual boundaries that determine the limits of [these] interactions.
[...]. The invisible boundaries that define niches are a complex topic, still only partly understood.

(ADAPTIVE) JUDGMENT

power of judging rightly and following the soundest course of action, based on knowledge, experience, understanding, …
Webster's New World College Dictionary

Aristotle's man of practical **wisdom**, the phronimos, does not ignore rules and models, or dispense
justice without criteria. He is observant of principles and, at the same time, open to their modification. He begins with nomoi – established law – and employs practical wisdom to determine how it should be applied in particular situations and when departures are warranted. Rules provide the guideposts for inquiry and critical reflection.

Leslie Paul Thiele: The Heart of Judgment Practical Wisdom, Neuroscience, and Narrative. Cambridge University Press 2006

JUDGMENT

DEDUCTION

INDUCTION

ABDUCTION

…

figures in:
explanation of behavior,
inference,
experience.

Hence the theory of
judgment has a place in:
psychology,
logic,
phenomenology.

Wayne M. Martin: Theories of Judgment. Psychology, Logic, Phenomenology. Cambridge Univ. Press (2006).

BEYOND THE TURING TEST
&
JUDGMENT

The Turing test, as originally conceived, focused on language and reasoning; **problems of perception and action were conspicuously absent**. The proposed tests will provide an opportunity to bring four important areas of AI research (language, reasoning, perception, and action) back into sync after each has regrettably diverged into a fairly independent area of research.

C. L. Ortitz Jr. Why we need a physically embodied Turing test and what it might look like.
AI Magazine 37 (2016) 55–62.

JUDGMENT

[Per Martin-Löf] explains what a **judgement** is from a constructivist point of view. The meaning of a judgement is fixed by laying down what it is that you must know in order to have the right to make the judgement in question. Starting with one of the basic judgemental forms *A is true*, where A is a proposition, we can say that A is true if there exists a verification of A , that is, if a proof of A has been constructed. We thus have obtained a **verification principle of** truth. […] the idea of a judging agent and that of an objective reason or ground play a central role in Martin-Löf's theory.

M.van der Schaar (ed.), Judgement and the Epistemic Foundation of Logic, Springer 2013, xiv

ADAPTIVE JUDGMENT

[…] a judgement is a piece of knowledge, and you have to clarify what knowledge.

Per Martin-Löf: Verificationism Then and Now.
In: M.van der Schaar (ed.), Judgement and the Epistemic Foundation of Logic, Springer 2013, 3-14

JUDGMENT

Reasoning of this kind is the least studied from the theoretical point of view and its structure is not sufficiently understood, in spite of many interesting theoretical research in this domain. The meaning of *common sense reasoning*, considering its scope and significance for some domains, is fundamental and rough set theory can also play an important role in it but more fundamental research must be done to this end.

Z. Pawlak, A. Skowron: Rudiments of rough sets. Information Sciences, 177(1):3-27, 2007

PRACTICAL JUDGMENT

Practical judgment is not algebraic calculation. Prior to any deductive or inductive reckoning, the judge is involved in selecting objects and relationships for attention and assessing their interactions. Identifying things of importance from a potentially endless pool of candidates, assessing their relative significance, and evaluating their relationships is well beyond the jurisdiction of reason

Leslie Paul Thiele: The Heart of Judgment Practical Wisdom, Neuroscience, and Narrative. Cambridge University Press 2006

ADAPTIVE JUDGMENT

<u>JUDGMENT</u>

is a reasoning process for reaching decisions or drawing conclusions under uncertainty, vagueness and/or imperfect knowledge performed by agents on complex granules

<u>ADAPTIVE JUDGMENT</u> is based on adaptive techniques for continuous judgement performance improvement.

ADAPTIVE JUDGMENT

- Searching for relevant approximation spaces
  - new features, feature selection
  - rule induction
  - measures of inclusion
  - strategies for conflict resolution
  - …
- Adaptation of measures based on the minimal description length: quality of approximation vs description length
- Reasoning about changes
- Selection of perception (action and sensory) attributes
- Adaptation of quality measures over computations relative to agents
- Adaptation of object structures
- Strategies for knowledge representation and interaction with knowledge bases
- Ontology acquisition and approximation
- Language for cooperation development and evolution
- …

COMPLEX GRANULES
IN
DEALING WITH PROBLEMS
BEYOND ONTOLOGIES

EVOLVING LANGUAGES
FOR PERCEIVING, REASONING AND ACTING
TOWARD ACHIEVING GOALS

***

RISK MANAGEMENT IN COMPLEX SYSTEMS

JUDGMENT IN APPROXIMATION OF LANGUAGES

JUDGMENT FOR ONTOLOGY APPROXIMATION

⇓

JUDGMENT FOR APPROXIMATION OF REASONING

GOTTFRIED WILHELM LEIBNIZ

[…] If controversies were to arise, there would be no more need of disputation between two philosophers than between two accountants. For it would suffice to take their pencils in their hands, and say to each other: *Let us calculate*.

[...] Languages are the best mirror of the human mind, and that a precise analysis of the signification of words would tell us more than anything else about the operations of the understanding.

Leibniz, G.W. : Dissertio de Arte Combinatoria (1666).
Leibniz, G.W.: New Essays on Human Understanding (1705), (translated by Alfred Gideon Langley, 1896), (Peter Remnant and Jonathan Bennett (eds.)). Cambridge University Press (1982).

COMPUTING WITH WORDS
LOTFI A. ZADEH

[…] Manipulation of perceptions plays a key role in human recognition, decision and execution processes. As a methodology, computing with words provides a foundation for a computational theory of perceptions - a theory which may have an important bearing on how humans make - and machines might make – perception - based rational decisions in an environment of imprecision, uncertainty and partial truth.

[…] computing with words, or CW for short, is a methodology in which the objects of computation are words and propositions drawn from a natural language.

Lotfi A. Zadeh1: From computing with numbers to computing with words – From manipulation of measurements to manipulation of perceptions. IEEE Transactions on Circuits and Systems 45(1), 105–119 (1999)

JUDEA PEARL- TURING AWARD 2011

for fundamental contributions to artificial intelligence through the development of a calculus for probabilistic and causal reasoning.

Traditional statistics is strong in devising ways of describing data and inferring distributional parameters from sample.

Causal inference requires two additional ingredients:

- *a science-friendly language for articulating causal knowledge*,

and

- *a mathematical machinery for processing that knowledge, combining it with data and drawing new causal conclusions about a phenomenon*.

Judea Pearl: Causal inference in statistics: An overview. Statistics Surveys 3, 96-146 (2009)

THE WITTGENSTEIN IDEA ON LANGUAGE GAMES

Wittgenstein, L.: Philosophical Investigations. (1953) (translated by G. E. M. Anscombe) (3rd Ed), Blackwell Oxford1967

JUDGMENT TO CONTROL COMPUTATIONS IN INTERACTIVE INTELLIGENT SYSTEMS (IIS)

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RISK MANAGEMENT AND COST/BENEFIT ANALYSIS IN IIS

Jankowski, A., Skowron, A., Wasilewski, P.: Interactive Computational Systems. CS&P 2012

Jankowski, A., Skowron, A., Wasilewski, P.: Risk Management and Interactive Computational Systems. Journal of Advanced Mathematics and Mathematics 2012

Jankowski, A.: Complex Systems Engineering: Conclusions from Practical Experience, Springer 2015, (in preparation)

HOW TO CONTROL COMPUTATIONS IN INTERACTIVE INTELLIGENT SYSTEMS (IIS) ?

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RISK MANAGEMENT IN IIS

Jankowski, A., Skowron, A., Wasilewski, P.: Interactive Computational Systems. CS&P 2012

Jankowski, A., Skowron, A., Wasilewski, P.: Risk Management and Interactive Computational Systems. Journal of Advanced Mathematics and Mathematics 2012

RISK IS THE EFFECT OF UNCERTAINTY ON OBJECTIVES (ISO 31K)

In practice risk management inference requires two additional ingredients
(slightly modified the Judea Pearl sentences):

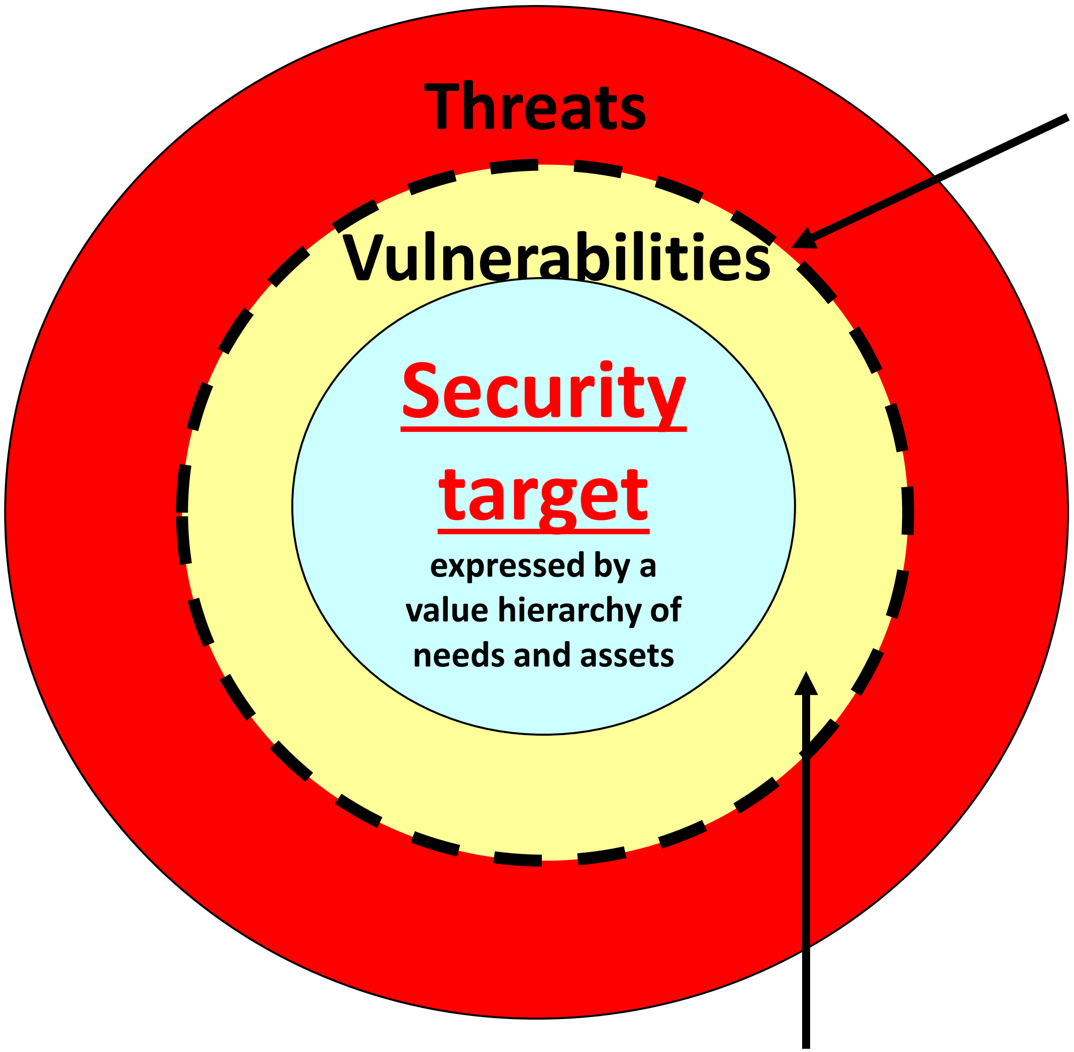
a science-friendly language for articulating risk management knowledge,

and

a mathematical machinery for processing that knowledge, combining it with data and drawing new risk management conclusions about a phenomenon.

THREATS AND VULNERABILITIES

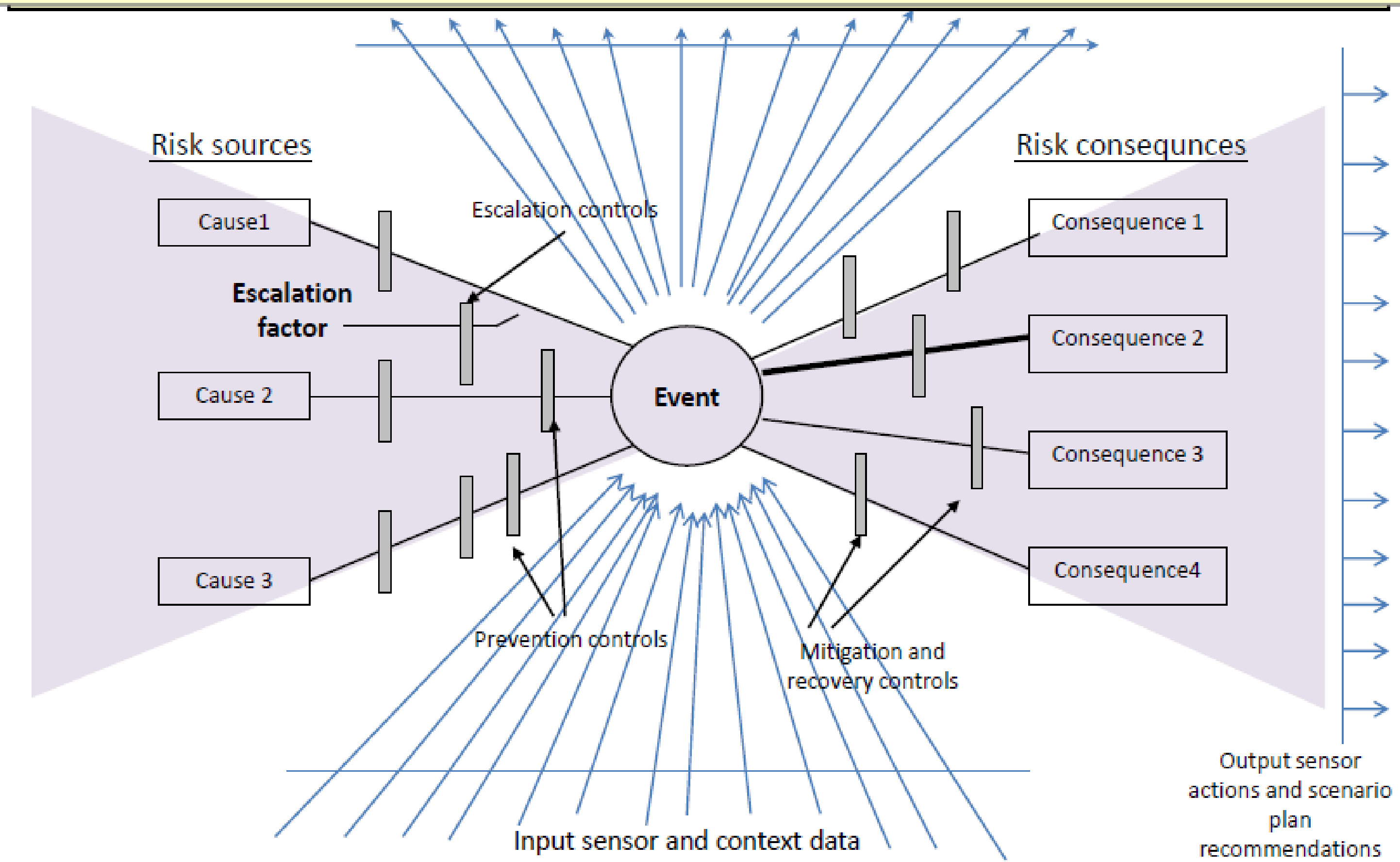
controls



vulnerabilities used by threats

EXAMPLE OF BOW TIE DIAGRAM FOR UNWANTED CONSEQUENCES

Wisdom = Interactions+ Adaptive Judgment + Knowledge
Adaptive Judgement = Adaptive Hierarchy of Needs & Values + Adaptive Linking + Adaptive Inference
Adaptive Inference = Inference + Inference Evaluation & Adaptation
Inference = Reasoning + Modelling + Assessment + Planning + …
Reasoning= Induction + Deduction + Abduction + …
Knowledge = Scope + Ontology + Rules of Language Use + Judged Language Expressions + Judged Adaptive Judgment Rules and Reasoning Schemes



EXTENSION:
RISK MANAGEMENT +
EFFICIENCY MANAGEMENT

PERCEPTION BASED COMPUTING

The main idea of this book is that perceiving is a way of acting. It is something we do. Think of a blind person tap-tapping his or her way around a cluttered space, perceiving that space by touch, not all at once, but through time, by skillful probing and movement. This is or ought to be, our paradigm of what perceiving is.

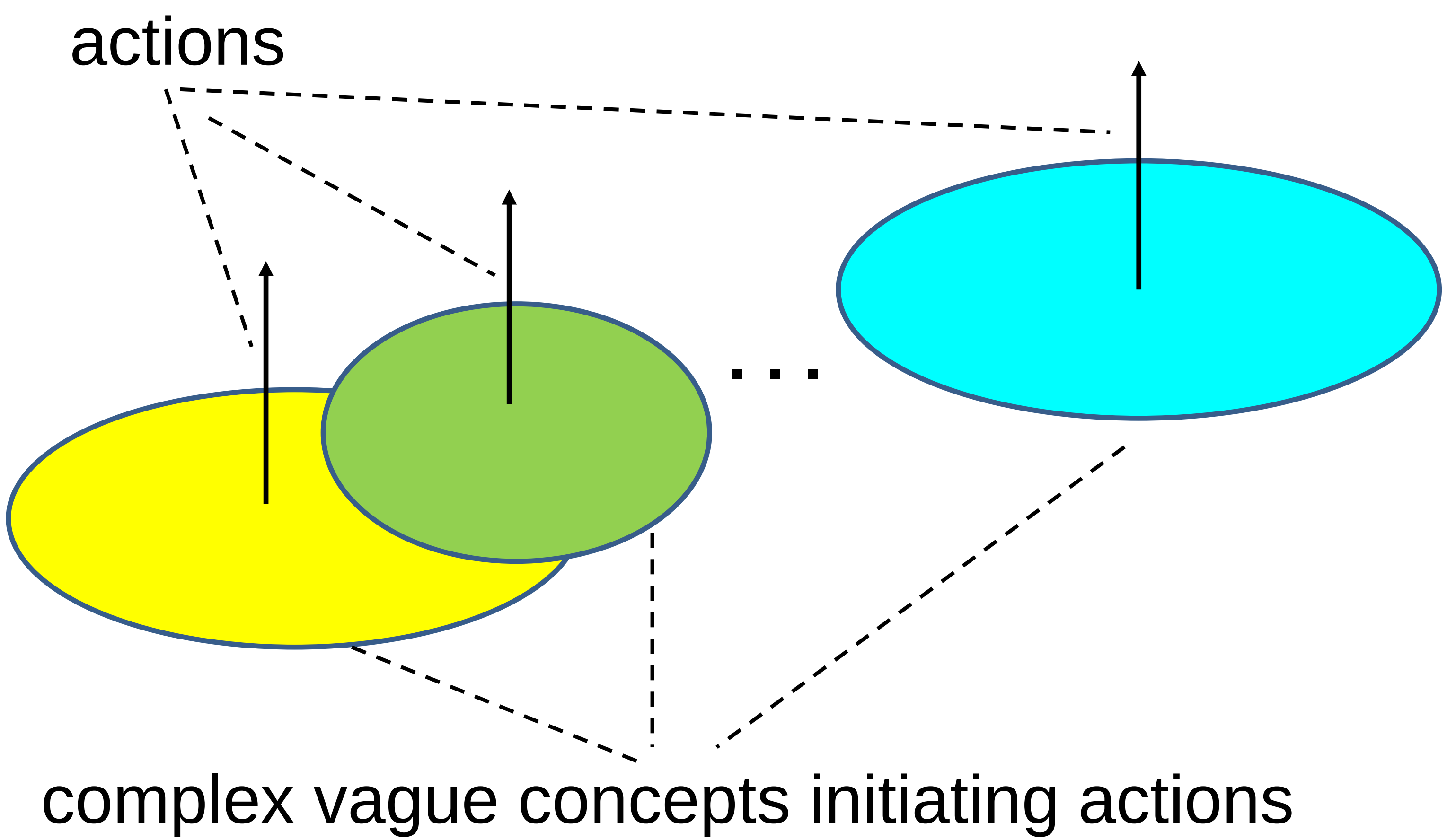
Alva Noë: Action in Perception, MIT Press 2004 interaction: agent → sensory and action attributes - only activated by agent attributes *A*(*t*) at time *t* are performing measurements and actions

history of sensory measurements and selected lower level actions over a period of time

	time	a_1	…	ac_1	…
x_1	1				
x_2	2				
…	…				

	features of histories	higher level action
…		

DISCOVERY OF COMPLEX GAMES OF INTERACTIONS



SUMMARY

THE ROLE OF RS IN INTERACTIVE GRANULAR COMPUTING IS AND WILL BE IMPORTANT

IN REAL LIFE APPLICATIONS WE ARE FORCED TO DEAL WITH MORE AND MORE COMPLEX VAGUE CONCEPTS.
DUE TO UNCERTAINTY THESE CONCEPTS CAN BE APPROXIMATED ONLY.

SUMMARY

INTERACTIVE COMPUTATIONS
ON
COMPLEX GRANULES

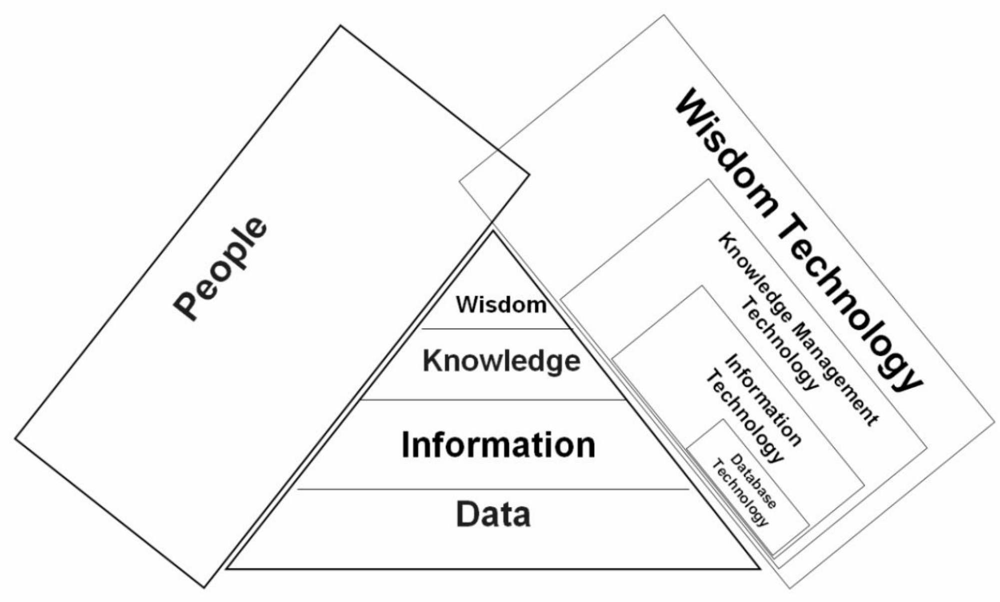
TOWARD RISK MANAGEMENT IN COMPLEX SYSTEMS

HUGE GAP BETWEEN
THE THEORY AND PRACTICE OF IMPLEMENTING COMPLEX SYSTEMS
(GAP PROBLEM)

Jankowski, A: Complex Systems Engineering: Wisdom for Saving Billions based on Interactive Granular Computing. Springer 2016 (in preparation)

WISDOM TECHNOLOGY (WisTech)

WISDOM=
INTERACTIONS +
ADAPTIVE
JUDGEMENT +
KNOWLEDGE
BASES



IGrC = systems based on interactive computations on complex granules with use of domain (expert) knowledge, process mining, concept learning, …

Jankowski, A. Skowron: A wistech paradigm for intelligent systems. Transactions on Rough Sets VI: LNCS Journal Subline, LNCS 4374, 2007, 94–132

International Rough Set Society http://www.roughsets.org

Group at Warsaw University:

http://logic.mimuw.edu.pl

RSES: http://logic.mimuw.edu.pl/~rses/

Rough Set Database System:

http://rsds.univ.rzeszow.pl/

RoughSets: Data Analysis Using Rough Set and Fuzzy Rough Set Theories (package in R)

https://cran.r-project.org/web/packages/RoughSets/index.html

Journal: Transactions on Rough Sets

http://roughsets.home.pl/www/index.php?option=com_content&task=view&id=14&Itemid=32

http://scholar.google.com/citations?user=fYu9ryIAAAAJ&hl=en&oi=ao

http://scholar.google.com/citations?user=zVpMZBkAAAAJ&hl=en&oi=ao

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Skowron, A., Suraj, Z. (eds.): Rough Sets and Intelligent Systems. Professor Zdzislaw Pawlak in Memoriam. Series Intelligent Systems Reference Library 42-43, Springer (2013).

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J. Kacprzyk, W. Pedrycz (eds.), Handbook of Computational Intelligence, Springer, 2015 (part on rough sets).

Jankowski, A., Complex Systems Engineering: Wisdom for Saving Billions based on Interactive Granular Computing. Springer 2016 (in preparation)

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Skowron, A., Rauszer, C., The discernibility matrices and functions in information systems, In: R. Słowiński (ed.), Intelligent Decision Support – Handbook of Applications and Advances of the Rough Sets Theory, System Theory, Knowledge Engineering and Problem Solving, vol. 11, pp. 331--362. Kluwer, Academic Publishers, Dordrecht, The Netherlands (1992).

THANK YOU !